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Abstract – We argue that the double-slit experiment can be understood much better by considering it as an experiment whereby one uses electrons to study the set-up rather than an experiment whereby we use a set-up to study the behaviour of electrons. We also show how Gödel's concept of undecidability can be used in an intuitive way to make sense of the double-slit experiment and the quantum rules for calculating coherent and incoherent probabilities. We meet here a situation where the electrons always behave in a fully deterministic way, while the detailed design of the set-up may render the question about the way they move through the set-up experimentally undecidable.

Introduction. – The double-slit experiment has been qualified by Feynman [1] as the only mystery of quantum mechanics (QM). Its mystery resides in an apparent paradox between the QM result and what we expect on the basis of our intuition. What we want to explain in this Letter is that this apparent paradox is a probability paradox. By this we mean that the paradox does not reside in some special property of the electron that could act both as a particle and a wave, but in the fact that we use two different definitions of probability in the intuitive approach and in the calculations. It is the difference between these two definitions which leads to the paradox, because the two definitions are just incompatible. In our discussion we will very heavily rely on the presentations by Feynman, even though further strange aspects have been pointed out by other authors later on, e.g. in the discussion of the delayed-choice experiment by Wheeler [2] and of the quantum eraser experiment [3], which can also be understood based on our discussion.

Feynman's essentials. – Feynman illustrates the paradox by comparing tennis balls and electrons. Tennis balls comply with classical intuition, while electrons behave according to the rules of QM. There is however, a small oversimplification in Feynman's discussion. He glosses over a detail, presumably for didactical reasons. When the electron behaves quantum mechanically and only one slit is open, the experiment will give rise to diffraction fringes, which can also not be understood in terms of a classical description in terms of tennis balls. But the hardest part of the mystery is that in the quantum mechanical regime we get a diffraction pattern when only one slit is open, while we get an interference pattern when both slits are open. This means that the single-slit probabilities even do not add up to an interference pattern when we allow for the quantum nature of the electron in a single-slit experiment. We will therefore compare most of the time the two quantum mechanical situations rather than electrons and tennis balls.

What Feynman describes very accurately is how quantum behaviour corresponds to the idea that the electron does not leave any trace behind in the set-up of its interactions with it. (We exclude here from our concept of a set-up the detectors that register the electrons at the very end of their history). We cannot tell with what part of the set-up the electron has interacted, because the interaction has been coherent. This corresponds to "wave behaviour". At the very same energy, a particle can also interact incoherently with the set-up and this will then result in classical "particle behaviour". The difference is that when the particle has interacted incoherently we do have the possibility to figure out its path trough the device, because the electron has left behind indications of its interactions with the measuring device within the device.

A nice example of this difference between coherent and incoherent interactions occurs in neutron scattering. In its interaction with the device, the neutron can flip its spin. The conservation of angular momentum implies then that there must be a concomitant change of the spin of a nucleus within an atom of the device. At least in principle the change of the spin of this nucleus could be detected by comparing the situations before and after the passage of the neutron, such that the history of the neutron could be reconstructed. Such an interaction with spin flip corresponds to incoherent neutron scattering. But the neutron can also interact with the atom without flipping its spin. There will be then no trace of the passage of the neutron in the form of a change of spin of a nucleus, and we will never be able to find out the history of the particle from a *post facto* inspection of the measuring device. An interaction without spin flip corresponds to coherent scattering. Note that this discussion only addresses the coherence of the spin interaction. There are other types of interaction possible and in order to have a globally coherent process none of these interactions must leave a mark of the passage of the neutron in the system that could permit us to reconstruct its history. An example of an alternative distinction between coherent and incoherent scattering occurs in the discussion of the recoil of the atoms of the device. A crystal lattice can recoil as a whole (coherent scattering). Alternatively, the recoil can just affect a single atom (incoherent scattering).

In incoherent scattering the electron behaves like a tennis ball. The hardest part of the mystery of the double-slit experiment is thus the paradox which occurs when we compare coherent scattering in the single-slit and in the double-slit experiment. Feynman resumed this mystery by asking: How can the particle know if the other slit is open or otherwise. In fact, as its interactions must be local the electron should not be able to sense if the other slit is open (see below).

Caveats. – Let us now leave our intuition for what it is and turn to QM. In a purely QM approach we could make the calculations for the three configurations. We could solve the wave equations for the single-slit and double-slit experiments:

$$\begin{array}{lll} -\frac{\hbar^2}{2m}\Delta\psi_1 + V_1(\boldsymbol{r})\,\psi_1 = -\frac{\hbar}{\imath}\frac{\partial}{\partial t}\psi_1, & \mathrm{S}_1 \text{ open}, & \mathrm{S}_2 \text{ closed}, \\ -\frac{\hbar^2}{2m}\Delta\psi_2 + V_2(\boldsymbol{r})\,\psi_2 = -\frac{\hbar}{\imath}\frac{\partial}{\partial t}\psi_2, & \mathrm{S}_1 \text{ closed}, & \mathrm{S}_2 \text{ open}, \\ -\frac{\hbar^2}{2m}\Delta\psi_3 + V_3(\boldsymbol{r})\,\psi_3 = -\frac{\hbar}{\imath}\frac{\partial}{\partial t}\psi_3, & \mathrm{S}_1 \text{ open}, & \mathrm{S}_2 \text{ open}. \end{array} \tag{1}$$

Here S_j refer to the slits. Within this theoretical framework we would still not obtain the result $|\psi_3|^2$ for the double-slit experiment by adding the probabilities $|\psi_1|^2$ and $|\psi_2|^2$ obtained from the solutions of the wave equations for the single-slit experiments. The fact that $|\psi_3|^2 \neq |\psi_1|^2 + |\psi_2|^2$ is at variance with our intuition about the rules of probability calculus in a way that seems to defy all our logic. Textbooks tell us that we should not add up probabilities but probability amplitudes, $|\psi_3|^2 = |\psi_1 + \psi_2|^2$. They describe this as the "superposition principle". They define wave functions $\psi = \sum_j c_j\chi_j$, and corresponding probabilities $p = |\psi|^2 = |\sum_j c_j\chi_j|^2$, whereby one must combine probability amplitudes rather than probabilities (coherent summing) in a linear way. They compare this to the addition of the amplitudes of waves like we can observe in a water tank, as also discussed by Feynman. But this is different from what we will define in this paper as a true superposition principle. A true superposition principle, based on the linearity of the equations would be that a linear combination $\psi = \sum_j c_j\chi_j$ is a solution of a Schrödinger equation:

$$-\frac{\hbar^2}{2m}\Delta\psi + V(\boldsymbol{r})\,\psi = -\frac{\hbar}{\imath}\frac{\partial}{\partial t}\psi. \tag{2}$$

when all wave functions χ_j are solutions of the *same* Schrödinger equation Eq. 2. This is then a straightforward mathematical result, and one can argue [4] that it leads to the probability rule $p = \sum_j |c_j|^2|\chi_j|^2$ (incoherent summing), whereby one combines probabilities $p_j = |\chi_j|^2$ in the classical way, which corresponds to common sense. But telling that the solution ψ_1 of a first equation with potential V_1 can be added to the solution ψ_2 of a second equation with a different potential V_2 to yield a solution ψ_3 for a third equation with a yet different potential V_3 can *a priori* not be justified by the mathematics and is not exact. It has nothing to do with the linearity of the equations. Summing the equations for ψ_1 and ψ_2 does not yield the equation for ψ_3. A solution of the wave equation for the single-slit experiment will not necessarily satisfy all the boundary conditions of the double-slit experiment, and vice versa. At the best, $\psi_3 = \psi_1 + \psi_2$ will in certain physical situations be a good approximation. But the fact that this is not rigorously exact and should be merely considered as a good numerical result rather than an exact physical truth is important. In fact, based on textbook presentations one could believe that it is an absolute physical truth in principle that one must replace the traditional rules of probability calculus $p_3 = p_1 + p_2$ by substituting the probabilities by their amplitudes. This is just not true. The belief must be vigorously eradicated because it leads to the misconception that there could exist a deep logical principle behind $\psi_3 = \psi_1 + \psi_2$, that in its proper context would be a truth that is as unshakable as $p_3 = p_1 + p_2$ in our traditional logic. As discussed below, mistaking the principle of substituting p by ψ for a deep mysterious absolute truth leads to insuperable conceptual problems in the case of destructive interference where $\psi_1(\boldsymbol{r}) + \psi_2(\boldsymbol{r}) = 0$.

To take this objection into account rigorously, we will define that the approximate solution $\psi_3 \approx \psi_1 + \psi_2$ of the double-slit wave equation follows a Huyghens' principle and note it as $\psi_3 = \psi_1 \boxplus \psi_2$, reserving the term superposition principle for the case when we combine wave functions that are all solutions of the same linear equation. We make this distinction between the superposition principle (with incoherent summing) and a Huyghens' principle (with coherent summing) to lay a mathematical basis for justifying that we have two different rules for calculating probabilities and that both the incoherent rule $p = \sum_j |c_j|^2|\chi_j|^2$ and the coherent rule $p = |\psi|^2 = |\sum_j c_j\chi_j|^2$ are correct within their respective domains of validity. This is the mathematical essence of the problem. QM just tells us that once we have an *exact* pure-state solution of a wave equation, we must square the amplitude of the wave function to obtain an *exact* probability distribution.

The very last thing we can do in face of a very hard paradox is to capitulate and think that we are not able to think straight. We will thus categorically refuse to yield to such defeatism. If we believe in logic, the rule $p_3 = p'_1 + p'_2$, where p'_1 and p'_2 are the probabilities to traverse the slits in the double-slit experiment, must still be exact. We are then compelled to conclude that in QM the probability p'_1 for traversing slit S_1 when slit S_2 is open is manifestly different from the probability p_1 for traversing slit S_1 when slit S_2 is closed. We can then ask with Feynman how the particle can know if the other slit is open or otherwise if its interactions are local.

Local interactions, non-local probabilities. – The solution to that problem is that *the interactions* of the electron with the device *are locally defined* while the probabilities defined by the wave function are not. *The probabilities are non-locally, globally defined.* When we follow our intuition, the electron interacts with the device in one of the slits. The corresponding probabilities are local interaction probabilities. We may take this point into consideration. Following our intuition we may then think that after doing so we are done. But in QM the story does not end here. The probabilities are globally defined and we must solve the wave function with the global boundary conditions. We may find locally a solution to the wave equation based on the consideration of the local interactions, but that is not good enough. The wave equation must also satisfy boundary conditions that are far away from the place where the electron is interacting. The QM probabilities are defined with respect to the global geometry of the set-up. This global geometry is fundamentally non-local, and the ensuing probability distribution is also non-locally defined. This claim may look startling. To make sense of it we propose the following slogan, which we will explain below: *"We are not studying electrons with the measuring device, we are studying the measuring device with electrons"*. This slogan introduces a paradigm shift that will grow to a leading principle as we go along. We can call it the holographic principle (see below).

In fact, we cannot measure the interference pattern in the double-slit experiment with one electron impact on a detector screen. We must make statistics of many electron impacts. We must thus use many electrons and measure a probability distribution for them. The probabilities must be defined in a globally self-consistent way. The definitions of the probabilities that prevail at one slit may therefore be subject to compatibility constraints imposed by the definitions that prevail at the other slit. We are thus measuring the probability distribution of an ensemble of electrons in interaction with the whole device. While a single electron cannot know if the other slit is open or otherwise, the ensemble of electrons will know it, because all parts of the measuring device will eventually be explored by the ensemble of electrons if this ensemble is large enough, *i.e.* if our statistics are good enough. When this is the case, the interference pattern will appear. Reference [5] gives actually a nice illustration of how the interference pattern builds up with time.

The geometry of the measuring device is non-local in the sense that a single electron cannot sense all aspects of the set-up through its local interactions. There is no contradiction with relativity in the fact that the probabilities for these local interactions must fit into a global probability scheme that is dictated also by parts of the set-up a single electron cannot probe. We must thus realize how Euclidean geometry contains information that in essence is non-local, because it cannot all be probed by a single particle, but that this is not in contradiction with the theory of relativity. The very Lorentz frames used to write down the Lorentz transformations are non-local because they assume that all clocks in the frame are synchronized up to infinite distance. It is by no means possible to achieve this, such that the very tool of a Lorentz frame conceptually violates the theory of relativity. But this remains without any practical incidence on the validity of the theory.

A classical analogy. – We can render these ideas clear by an analogy. Imagine a country that sends out spies to an enemy country. The electrons behave as this army of spies. The double-slit set-up is the enemy country. The physicist is the country that sends out the spies. Each spy is sent to a different part of the enemy's country, chosen by a random generator. They will all take photographs of the part of the enemy country they end up in. The spies may have an action radius of only a kilometer. Some of the photographs of different spies will overlap. These photographs correspond to the spots left by the electrons on your detector. If the army of spies you send out is large enough, then in the end the army will have made enough photographs to assemble a very detailed complete map of the country. That map corresponds to the interference pattern. In assembling the global map from the small local patches presented by the photographs we must make sure that the errors do not accumulate such that everything fits together self-consistently. This is somewhat analogous with the boundary conditions of the wave function that must be satisfied globally, whereby we can construct the global wave function also by assembling patches of local solutions. The tool one can use to ensure this global consistency is a Huyghens' principle. An example of such a Huyghens' principle is Feynman's path integral method or Kirchhoff's method in optics. The principle is non-local and is therefore responsible for the fact that we must carry out calculations that are purely mathematical but have no real physical meaning. They may look incomprehensible if we take them literally, because they may involve e.g. backward propagation in space and even in time [6, 7].

The interference pattern presents this way the information about the whole experimental set-up. It does not present this information directly but in an equivalent way, by an integral transform. This can be seen from Born's treatment of the scattering of particles of mass m_0 by a potential V_s, which leads to the differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{m_0}{4\pi^2}\,|\mathscr{F}(V_s)(\boldsymbol{q})|^2, \tag{3}$$

where $\boldsymbol{p} = \hbar\boldsymbol{q}$ is the momentum transfer. The integral transform is here the Fourier transform $\mathscr{F}$, which is a even a one-to-one mapping. This result is derived within the Born approximation and is therefore an approximate result. In a more rigorous setting, the integral transform could be e.g. the one proposed by Dirac [8], which Feynman was able to use to derive the Schrödinger equation [9]. The Huyghens' principles used by Feynman and Kirchhoff are derived from integral transforms to which they correspond. In a double-slit experiment, V_s embodies just the geometry of the set-up. Combined with a reference beam $\mathscr{F}(V_s)(\boldsymbol{q})$ yields its hologram. The spies in our analogy are not correlated and not interacting, but the information about the country is correlated: It is the information we put on a map. The map will e.g. show correlations in the form of long straight lines, roads that stretch out for thousands of miles, but none of your spies will have seen these correlations and the global picture. They just have seen the local picture of the things that were situated within their action radius. The global picture, the global information about the enemy country is non-local, and contains correlations, but it can nevertheless be obtained if you send out enough spies to explore the whole country, and it will show on the map assembled. That is what we are aiming at by invoking the non-locality of the Lorentz frame and the non-locality of the wave function. The global information gathered by many electrons contains the information how many slits are open. It is that kind of global information about your set-up that is contained in the wave function. You need many single electrons to collect that global information. A single electron just gives you one impact on the detector screen. That is almost no information. Such an impact is a Dirac delta measure, derived from the Fourier transform of a flat distribution. It contains hardly any information about the set-up because it does not provide any contrast. This global geometry contains thus more information than any single electron can measure through its local interactions. And it is here that the paradox creeps in. The probabilities are not defined locally, but globally. The interactions are local and in following our intuition, we infer from this that the definitions of the probabilities will be local as well, but they are not. The space wherein the electrons travel in the double-slit experiment is not simply connected, which is, as we will see, a piece of global, topological information apt to profoundly upset the way we must define probabilities.

Highly simplified descriptions still catch the essence. – The description of the experimental set-up we use to calculate a wave function is conventionally highly idealized and simplified. Writing an equation that would make it possible to take into account all atoms of the macroscopic device in the experimental set-up is a hopeless task. Moreover, the total number of atoms in "identical" experimental set-ups is only approximately identical. In such a description there is no thought for the question if the local interaction of a neutron involves a spin flip or otherwise. Despite its crudeness, such a purely geometrical description is apt to seize a crucial ingredient of any experiment whereby interference occurs. It is able to account for the difference between set-ups with one and two slits, as in solving the wave equation we unwittingly avoid the pitfall of ignoring the difference between globally and locally defined probabilities, rendering the solution adopted tacitly global. In this sense the probability paradox we are confronted with is akin to Bertrand's paradox in probability calculus. It is not sufficient to calculate the interaction probabilities locally. We must further specify how we will use these probabilities later on in the procedure to fit them into a global picture. The probabilities will be only unambiguously defined if we define simultaneously the whole protocol we will use to calculate with them.

Winnowing out the over-interpretations. – It is now time to get rid of the particle-wave duality. Electrons are always particles, never waves. As pointed out by Feynman, electrons are always particles because a detector detects always a full electron at a time, never a fraction of an electron. Electrons never travel like a wave through both slits simultaneously. But in a sense, their probability distribution does. It is the probability distribution of many electrons which displays wave behaviour and acts like a flowing liquid, not the individual electrons themselves. This postulate only reflects literally what quantum mechanics says, viz. that the wave function is a probability amplitude, and that it behaves like a wave because it is obtained as the solution of a wave equation. Measuring the probabilities requires measuring many electrons, such that the probability amplitude is a probability amplitude of an ensemble of electrons. Although this sharp dichotomy is very clearly present in the rules, we seem to loose sight of it when we are reasoning intuitively. This is due to a tendency towards *"Hineininterpretierung"* in terms of Broglie's initial idea that the particles themselves, not their probability distributions, would be waves. These heuristics have historically been useful but are reading more into the issue than there really is. Their addition blurs again the very accurate sharp pictures provided by QM. With hindsight, we must therefore dispense with the particle-wave duality. The rules of QM are clear enough in their own right: *In claro non interpretatur!* Wave functions also very obviously do not collapse. They serve to describe a statistical ensemble of possible events, not outcomes of single events.

It is also time to kill the traditional reading of $\psi_3 = \psi_1 \boxplus \psi_2$ in terms of a "superposition principle", based on the wave picture. It is is only a convenient numerical recipe, a Huyghens' principle without true physical meaning. We can make the experiment in such a way that only one electron is emitted by the source every quarter of an hour. Still the interference pattern will build up if we wait long enough. But if ψ_1 and ψ_2 were to describe the correct probabilities from slit S_1 and slit S_2, we would never be able to explain destructive interference. How could a second electron that travels through slit S_2 erase the impact made on the detector screen of an electron that traveled through slit S_1 hours earlier? We may speculate that the electron feels whether the other slit is open or otherwise. E.g. the electron might polarize the charge distribution inside the measuring device and the presence of the other slit might influence this induced charge distribution. This would be an influence at a distance that is not incompatible with the theory of relativity. But this scenario is not very likely. As pointed out by Feynman interference is a universal phenomenon. It exists also for photons, neutrons, helium atoms, etc... We already capture the essence of this universal phenomenon in a simple, crude geometrical description of the macroscopic set-up of the experiment. While this could be a matter of pure luck according to the principle that fortune favors fools, it is not likely that one could translate the scenario evoked for electrons to an equivalent scenario in all these different situations. E.g. how could the fact that another slit is open (in a nm-sized double-slit experiment) affect the nuclear process at the fm scale of the spin flip of a neutron? The generality of the scenario based on an influence at a distance is thus not very likely.

We must thus conclude that $\psi_3 = \psi_1 \boxplus \psi_2$ is a very good numerical approximation for the true wave function ψ_3', whereby the physically meaningful identity reads $\psi_3' = \psi_1' + \psi_2'$ in terms of other wave functions ψ_1' and ψ_2'. The wave functions ψ_1' and ψ_2' must now both be zero, $\psi_1'(\boldsymbol{r}) = \psi_2'(\boldsymbol{r}) = 0$, in all places $\boldsymbol{r}$ where we have "destructive interference", because $p_3 = p_1 + p_2$ must still be valid. In other words $\psi_1 \neq \psi_1'$ and $\psi_2 \neq \psi_2'$.

Undecidability. – We can further improve our intuition for this by another approach based on undecidablity. Questions that are undecidable are well known in mathematics. Examples occur e.g. in Gödel's theorem [10]. The existence of such undecidable questions may look hilarious to common sense but this does not need to be. In fact, the reason for the existence of such undecidable questions is that the set of axioms of the theory is incomplete. We can complete then the theory by adding an axiom telling the answer to the question is "yes", or by adding an axiom telling the answer to the question is "no". The two alternatives permit to stay within a system based on binary logic (*"tertium non datur"*) and lead to two different axiomatic systems and thus to two different theories. An example of this are Euclidean and hyperbolic geometry [11]. In Euclidean geometry one has added on the fifth parallels postulate to the first four postulates of Euclid, while in hyperbolic geometry one has added on an alternative postulate that is at variance with the parallels postulate. We are actually not forced to make a choice: We can decide to study a "pre-geometry", wherein the question remains undecidable. The axiom one has to add can be considered as information that was lacking in the initial set of four axioms. Without adding it one cannot address the yes-or-no question which reveals that the axiomatic system without the parallels postulate added is incomplete. As Gödel has shown, we will almost always run eventually into such a problem of incompleteness. On the basis of Poincaré's mapping between hyperbolic and Euclidean geometry [11], we can appreciate which information was lacking in the first four postulates. The information was not enough to identify the straight lines as really straight, as we could still interpret the straight lines in terms of half circles in a half plane.

When the interactions are coherent in the double-slit experiment, the question through which one of the two slits the electron has traveled is very obviously also undecidable. Just like in mathematics, this is due to lack of information. We just do not have the information that could permit us telling which way the electron has gone. This is exactly what Feynman pointed out so carefully. In his lecture he considers three possibilities for our observation of the history of an electron: "slit S_1", "slit S_2", and "not seen". The third option corresponds exactly to this concept of undecidability. He works this out with many examples in reference [1], to show that there is a one-to-one correspondence between undecidability and coherence. Coherence already occurs in a single-slit experiment, where it is at the origin of the diffraction fringes. But in the double-slit experiment the lack of knowledge becomes all at once amplified to an objective undecidability of the question through which slit the electron has traveled, which does not exist in the single-slit experiment. What happens here in the required change of the definition of the probabilities has nothing to do with a change in local physical interactions. It has only to do with the question how we define a probability with respect to a body of available information. The probabilities are in a sense conditional because they depend on the information available. As the lack of information is different in the double-slit experiment, the body of information available changes, such that the probabilities must be defined in a completely different way. According to common-sense intuition whereby we reason only on the local interactions, opening or closing the other slit would not affect the probabilities or only affect them slightly, but this is wrong. We may also think that the undecidability is just experimental such that it would not matter for performing our probability calculus. We may reckon that in reality, the electron must have gone through one of the two slits anyway. We argue then that we can just assume that half of the electrons went one way, and the other half of the electrons the other way, and that we can then use statistical averaging to simulate the reality, just like we do in classical statistical physics. We can verify this argument by detailed QM calculations. We can calculate the solutions of the three wave equations in Eq. 1 and compare $|\psi_3|^2$ with the result of our averaging procedure based on $|\psi_1|^2$ and $|\psi_2|^2$. This will reproduce the disagreement between the experimental data and our classical intuition, confirming QM is right.

To make sense of this we may point out that we are not used to logic that allows for undecidability. Decided histories with labels S_1 or S_2 occur in a theory based on a system of axioms $\mathcal{A}_1$ (binary logic), while the undecided histories occur in a theory based on an all together different system of axioms $\mathcal{A}_2$ (ternary logic). In fact, the averaging procedure is still correct in $\mathcal{A}_2$ because the electron travels indeed either through S_1 or through S_2 following binary logic. But the information we obtain about the electron's path does not follow binary logic. It follows ternary logic. Information biases probabilities, which is why insurance companies ask their clients to fill forms requesting information about them. Due to the information bias the probabilities $|\psi'_1|^2$ and $|\psi'_2|^2$ to be used in $\mathcal{A}_2$ are very different from the probabilities $|\psi_1|^2$ and $|\psi_2|^2$ to be used in $\mathcal{A}_1$. The paradox results thus from the fact that we just did not imagine that such a difference could exist. Assuming $\psi'_j = \psi_j$, for $j = 1, 2$ amounts to neglecting the ternary bias of the information contained in our data and reflects the fact that we are not aware of the global character of the definition of the probabilities. To show that the intuition $\psi'_j = \psi_j$, for $j = 1, 2$ is wrong, nothing is better than giving a counterexample. The counterexample is the double-slit experiment where clearly the probability is not given by $p_3 = |\psi_1|^2 + |\psi_2|^2$ but by $p_3 = |\psi'_1|^2 + |\psi'_2|^2 \approx |\psi_1 \boxplus \psi_2|^2$, where the index 3 really refers to the third (undecidable) option. It is then useless to insist any further.

The undecidability criterion corresponds to a global constraint that has a spectacular impact on the definition of the probabilities. The probabilities are *conditional* and *not absolute*. They are *physically* defined by the physical information gathered from the interactions with the set-up, *not absolutely* by some absolute divine knowledge about the path the electron has taken. The set-up biases the information we can obtain about that divine knowledge by withholding a part of the information about it. Einstein is perfectly right that the Moon is still out there when we are not watching. But we cannot find out that the Moon is there if we do not register any of its interactions with its environment, even if it is there. If we do not register any information about the existence of the Moon, then the information contained in our experimental results must be biased in such a way that everything looks as though the Moon were not there. Therefore, in QM the undecidability must affect the definition of the probabilities and bias them, such that $p'_j \neq p_j$, for $j = 1, 2$. The experimental probabilities must reflect the undecidability. In a rigorous formulation, this undecidability becomes a consequence of the fact that the wave function must be a function, because it is the integral transform of the potential, which must represent all the information about the set-up and its built-in undecidability. As the phase of the wave function corresponds to the spin angle of the electron, even this angle is thus uniquely defined. The way the electron travels through the set-up from a point $\boldsymbol{r}_1$ to a point $\boldsymbol{r}_2$ can thus have no incidence whatsoever on the phase of the wave function in $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. The exact application of this idea is worked out in reference [12], pp. 329-333, and depends critically on the fact that the space traversed by the electrons that end up in the detector is not simply connected. In an alternative approach we can try to revert this argument, and start from the idea of undecidability instead. As we describe the experiment in a purely geometrical way without any reference to spin flips, atomic recoils or other specific physical processes, the only way to account for the undecidability is imposing a left-right symmetry on the wave function. This highlights that the paradox is a probability paradox and is the reason why this wave function must mathematically be given by the symmetrical linear combination $\psi'_3 = \psi'_1 + \psi'_2$. It is just the symmetry-adapted wave function. For these reasons ψ'_1 and ψ'_2 must be weighted reductions of ψ'_3 to their respective slits, and be different from ψ_1 and ψ_2, whereby "accidentally" $\psi'_1 + \psi'_2 = \psi_1 + \psi_2$. The other symmetry-adapted combination is $\psi'_1 - \psi'_2$, which is antisymmetrical. There is a pitfall in the reversed approach which we automatically avoid in the direct approach. It is not obvious how we rule out intuitively $\psi'_1 - \psi'_2$, because we might have the intuition that one cannot measure spin angles or phases. As the only quantities we can measure then are probabilities, only the probabilities must be undecidable and we could therefore object that rejecting $\psi'_1 - \psi'_2$ is not cogent. The solution of this riddle is that the undecidability implies that $\psi'_1 - \psi'_2 = 0$. When we perform the change of basis to the symmetry-adapted functions, we must resist the temptation to use the functions ψ_1 and ψ_2, because they have nothing to do with the double-slit experiment. We cannot consider $\psi_1 - \psi_2 \neq 0$ as the basis vector that is complementary to the basis vector $\psi_3 = \psi_1 \boxplus \psi_2$, because the calculation $\psi_3 = \psi_1 \boxplus \psi_2$ is conceptually meaningless. Only the quantities ψ'_1, ψ'_2, $\psi'_1 - \psi'_2 = 0$ and $\psi'_1 + \psi'_2$ are physically meaningful. We see thus that $p_3 = |\psi'_1|^2 + |\psi'_2|^2$ and that interference does not exist physically. Interference has no meaning beyond the purely mathematical context of Huyghens' principle. Furthermore, the spin angle is conceptually a meaningful physical quantity. If it really could not manifest itself in any experiment, we would not need wave functions and it would imply that what counts in QM is more than just what can be observed. The assumption that we cannot measure spin angles must therefore be considered with extreme caution.

The reason why we can calculate $\psi'_3 = \psi'_1 + \psi'_2$ as $\psi_3 = \psi_1 \boxplus \psi_2$ can within the Born approximation be explained [4] by the linearity of the Fourier transform used in Eq. 3, which is a better argument than invoking the linearity of the wave equation. The reason for the presence of the Fourier transform in the formalism is the fact that the electron spins [4,12]. One can derive the whole wave formalism purely classically, just from the assumption that the electron spins. Eq. 3 hinges also crucially on the Born rule $p = |\psi|^2$. There is no rigorous proof for this rule but there exists ample justification for it [4]. The undecidability is completely due to the properties of the potential which defines both the local interactions and the global symmetry. But to highlight the importance of the undecidability we can reformulate the algebra by stating that the integral transform we must use for undecidable problems is the undecidable Fourier transform $\int_{\mathbb{R}^3} V(\boldsymbol{r}) \cos \boldsymbol{q} \cdot \boldsymbol{r}\, d\boldsymbol{r}$ rather than the decidable Fourier transform $\int_{\mathbb{R}^3} V(\boldsymbol{r})\, e^{\imath \boldsymbol{q} \cdot \boldsymbol{r}} d\boldsymbol{r}$.

Conclusion. – In summary, we have proposed an intelligible solution for the paradox of the double-slit experiment. It is perhaps not what everybody would call intuitive and it is not an absolutely rigorous mathematical proof, but it is logically intelligible and plausible. A much more detailed account of this work is given in reference [4], where we explain also how a similar analysis can be applied to paradoxes related to Bell-type inequalities.

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REFERENCES

[1] R.P. Feynman, R. Leighton, and M. Sands, in *The Feynman Lectures on Physics, Vol. 3*, Addison-Wesley, Reading, MA (1970). See also the video entitled *"Probability & uncertainty - the quantum mechanical view of nature"* at https://www.youtube.com/watch?v=2mIk3wBJDgE.

[2] J.A. Wheeler in *Mathematical Foundations of Quantum Theory*, A.R. Marlow editor, Academic Press (1978), pp. 9-48; J.A. Wheeler and W.H. Zurek, in *Quantum Theory and Measurement* (Princeton Series in Physics).

[3] Y.-H. Kim, R. Yu, S.P. Kulik, Y. Shih et M.O. Scully, Phys. Rev. Lett. **84**, 1-5 (2000).

[4] G. Coddens, CEA-01269569 (2016).

[5] A. Tonomura, J. Endo, T. Matsuda and T. Kawasaki, Am. J. Phys. **57**, 117 (1989); see also M.P. Silverman in *More than one Mystery, Explorations in Quantum Interference*, Springer, p. 3 (1995).

[6] R.P. Feynman, *QED, The Strange Theory of Light and Matter*, Princeton University Press (1988).

[7] J. Cramer, Rev. Mod. Phys. **58**, pp. 647-688, (1986).

[8] P.A.M. Dirac, Physikalishe Zeitschrift der Sowjetunion, Band 3, Heft 1 (1933).

[9] R.P. Feynman in *The Character of Physical Law*, MIT Press (1967).

[10] K. Gödel, in *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, Dover, New York, (1992).

[11] J. Dieudonné, in *Pour l'honneur de l'esprit humain - les mathématiques aujourd'hui*, Hachette, Paris (1987).

[12] G. Coddens, in *From Spinors to Quantum Mechanics*, Imperial College Press, (2015).