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Quantum Corrections in String Compactifications on SU(3) Structure Geometries

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ABSTRACT

We investigate quantum corrections to the classical four-dimensional low-energy effective action of type II string theory compactified on $SU(3)$ structure geometries. Various methods previously developed for Calabi–Yau compactifications are adopted to constrain – under some simple assumptions about the low-energy degrees of freedom – the leading perturbative corrections to the moduli space metrics in both α' and the string coupling constant. We find that they can be parametrized by a moduli dependent function in the hypermultiplet sector and a constant in the vector multiplet sector. We argue that under specific additional assumption they take – in complete analogy to the Calabi–Yau case – a universal form which depends only on the Euler characteristic of the six-dimensional compact space.

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1 Introduction

In this paper we consider type II string theory in backgrounds $M_{1,3} \times X_6$, where $M_{1,3}$ is a four-dimensional ($d = 4$) Lorentzian space-time and X_6 is a compact six-dimensional manifold. Demanding that the effective theory in $M_{1,3}$ admit eight unbroken supercharges ($\mathcal{N} = 2$ in $d = 4$) constrains X_6 to be a manifold with $SU(3)$ structure [1, 2].¹ Manifolds with $SU(3)$ structure admit a globally defined nowhere-vanishing spinor and as a consequence the structure group of the tangent space is reduced, meaning that it can be patched using only an $SU(3)$ subgroup of $SO(6)$. If in addition the spinor is covariantly constant the Levi-Civita connection has $SU(3)$ holonomy, and in this case X_6 is a Calabi-Yau manifold.

The $\mathcal{N} = 2$ low-energy effective Lagrangian of such backgrounds has been computed in refs. [2, 8–16] at the string tree level and for “large” manifolds X_6 assuming a suitable Kaluza-Klein reduction. For Calabi-Yau manifolds this is straightforward in that the massless modes are in one-to-one correspondence with the cohomology of X_6 , and the resulting $\mathcal{N} = 2$ supergravity is ungauged with no potential [17]. In the generalized case the corresponding analysis is much harder and can only be performed if there is a suitable hierarchy of the low-energy modes. In this case the effective action is a gauged $\mathcal{N} = 2$ supergravity with a potential which stabilizes (some of) the moduli.

The low-energy effective Lagrangian is corrected by higher-order α' terms as well as by string loops (g_s corrections). Both types of corrections can be parametrized by a scalar field (or a modulus) in the low-energy effective action. Higher-order α' corrections arise when the size of the manifold is comparable to the string length and thus they are controlled by the volume modulus of X_6 . String loop corrections on the other hand are counted by g_s with the dilaton being the corresponding modulus. For Calabi-Yau compactifications both types of corrections have been partially computed, and we review some of these results in the following. However, for generalized compactifications little is known about these corrections primarily due to the fact that they have no direct worldsheet description.

The purpose of this paper is to investigate what can be said about α' and g_s corrections in the specific case of $SU(3)$ structure compactifications. We will not focus on a particular background but rather attempt to constrain the generic form of the possible corrections, under some simple assumptions. We also do not consider any non-trivial NSNS or RR fluxes, or more general $SU(3) \times SU(3)$ structures (see footnote 1).

Using simple extensions of the arguments employed for Calabi-Yau compactifications in [18–23] we find – given there is an expansion in terms of a finite set of low-energy

¹More generally, viewed as generalized geometries [3–5], $\mathcal{N} = 2$ supersymmetry requires an $SU(3) \times SU(3)$ structure on the generalized tangent space [6–9] or even more generally an $SU(6)$ structure on the exceptional generalized tangent space [10].

fields – that the $\mathcal{N} = 2$ supersymmetry strongly constrains the leading corrections to the kinetic terms. In the hypermultiplet sector a non-renormalization theorem holds and the metric is perturbatively corrected only at one-loop by a moduli dependent function. In the type IIA vector multiplet sector we find α' corrections which in a large volume expansion correct the holomorphic prepotential by a constant term at leading order. In type IIB the vector multiplet sector on the other hand is not corrected at all. With the additional assumption of a putative mirror symmetry we are able to argue that all corrections are proportional to the Euler characteristic of X_6 exactly as in Calabi-Yau compactifications.

Generically, $SU(3)$ structure compactifications admit fewer Peccei–Quinn (PQ) symmetries than Calabi–Yau compactifications, since some of the corresponding moduli acquire a mass from the gauging of the $\mathcal{N} = 2$ supergravity. Remarkably, we find that the leading kinetic energy corrections still have the full set of PQ symmetries, even under shifts of the massive moduli.

The paper is organized as follows: In section 2 we review some basic facts about $\mathcal{N} = 2$ supergravity and $SU(3)$ structure compactifications. We also summarize our assumptions about the moduli space of low-energy fields. Section 3 then discusses the form of the leading corrections to the metrics on these moduli spaces: the prepotential for the vector multiplet fields, and the quaternionic-Kähler metric for the hypermultiplet fields. We first discuss the constraints on the metric of the geometric moduli that arise from dimensionally reducing the known ten-dimensional R^4 correction terms. We then turn to the full vector and hypermultiplet moduli space metrics, first for the α' corrections and then for the g_s corrections. Section 4 summarizes our results and contains some concluding remarks.

2 Preliminaries

2.1 $\mathcal{N} = 2$ supersymmetry in $M_{1,3}$

In order to discuss the α' and g_s corrections we need to briefly assemble some facts about $\mathcal{N} = 2$ supergravity (for a review see e.g. [24]). A generic spectrum contains a gravitational multiplet, n_v vector multiplets and n_h hypermultiplets.² The gravitational multiplet contains the spacetime metric, two gravitini and the graviphoton A_μ^0 . A vector multiplet contains a vector A_μ , two gaugini and a complex scalar t . Finally, a hypermultiplet contains two hyperini and four real scalars (q^1, q^2, q^3, q^4) .

The $\mathcal{N} = 2$ supersymmetry enforces the scalar field space M locally to be a product

$$M = M_v \times M_h , \tag{2.1}$$

where M_v is a $2n_v$ -dimensional manifold spanned by the complex scalars $t^i, i = 1, \dots, n_v$,

²Tensor multiplets are dual to hypermultiplets if they are massless and to vector multiplets if they are massive.

contained in n_v vector multiplets, while M_h is a $4n_h$ -dimensional manifold spanned by the real scalars q^u , $u = 1, \dots, 4n_h$, in n_h hypermultiplets. Thus their sigma-model Lagrangian is of the form

$$\mathcal{L} = g_{i\bar{j}}(t, \bar{t}) D_\mu t^i D^\mu \bar{t}^{\bar{j}} + h_{uv}(q) D_\mu q^u D^\mu q^v - V(t, \bar{t}, q) , \quad \mu = 0, \dots, 3 , \quad (2.2)$$

where $g_{i\bar{j}}$ is the metric on M_v while h_{uv} is the metric on M_h . In general, isometries on M_v and M_h can be gauged, so that D is an appropriate covariant derivative which includes the couplings of the charged scalars to the vector bosons. Only if the theory is gauged can it admit a non-trivial potential V , which furthermore is completely determined by the choice of gauging.

Supersymmetry further constrains M_v to be a special-Kähler manifold, so that the metric can be written as [25, 26]

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K^v \quad \text{with} \quad K^v = -\ln i \left(\bar{X}^I \mathcal{F}_I - X^I \bar{\mathcal{F}}_I \right) . \quad (2.3)$$

Both $X^I(t)$ and $\mathcal{F}_I(t)$, $I = 0, 1, \dots, n_v$, are holomorphic functions of the scalars t^i , and $\mathcal{F}_I = \partial \mathcal{F} / \partial X^I$ is the derivative of a holomorphic prepotential $\mathcal{F}(X)$ homogeneous of degree two. Furthermore, it is possible to adopt a system of ‘special coordinates’ in which $X^I = (i, t^i)$ (see e.g. [26] for further details).

M_h is similarly constrained to be a quaternionic-Kähler manifold, which means its holonomy group is of the form $Sp(1) \times H$ with $H \subset Sp(n_h)$ [27, 28]. There is a special class of quaternionic-Kähler manifolds which arise in string tree-level effective actions known as ‘special quaternionic-Kähler manifolds’ $M_{\text{SQK}}^{4n_h}$. These manifolds can be viewed as a $2n_h$ -dimensional torus fibred over a special-Kähler base, of the form $[SU(1, 1)/U(1)] \times M_{\text{SK}}^{2n_h-2}$, where $M_{\text{SK}}^{2n_h-2}$ is a special-Kähler manifold of dimension $2n_h - 2$. This relation between special-Kähler and quaternionic Kähler manifolds is known as the ‘c-map’ [29]

$$\frac{SU(1, 1)}{U(1)} \times M_{\text{SK}}^{2n_h-2} \mapsto M_{\text{SQK}}^{4n_h} . \quad (2.4)$$

The metric on $M_{\text{SQK}}^{4n_h}$ takes an explicit form known as the Ferrara–Sabharwal metric, which reads [30]

$$\begin{aligned} \mathcal{L} = & (\partial_\mu \phi)^2 + e^{4\phi} (\partial_\mu \sigma - \xi^I \partial_\mu \tilde{\xi}_I)^2 + g_{a\bar{b}} \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} \\ & - e^{2\phi} (\mathcal{N} + \bar{\mathcal{N}})^{IJ} (\partial_\mu \tilde{\xi}_I - 2i \mathcal{N}_{IK} \partial_\mu \xi^K) (\partial^\mu \tilde{\xi}_J + 2i \bar{\mathcal{N}}_{JL} \partial^\mu \xi^L) , \end{aligned} \quad (2.5)$$

where ϕ and σ span the $SU(1, 1)/U(1)$ factor of (2.4), ξ^I and $\tilde{\xi}_I$ are real coordinates of the torus fibre, and z^a are complex coordinates on the special Kähler base $M_{\text{SK}}^{2n_h-2}$.³ The

³In (2.5) we display the ungauged metric. If some of its isometries are gauged the corresponding ordinary derivatives are replaced by appropriate covariant derivatives.

metric $g_{a\bar{b}}(z, \bar{z})$ on $M_{\text{SK}}^{2n_h-2}$ satisfies (2.3) and thus can be characterized by a holomorphic prepotential $\mathcal{G}(z)$.⁴ The couplings $\mathcal{N}_{IJ}(z, \bar{z})$ are given in terms of $\mathcal{G}$ as

$$\mathcal{N}_{IJ} = -i\bar{\mathcal{G}}_{IJ} + 2 \frac{\text{Im } \mathcal{G}_{IK} z^K \text{Im } \mathcal{G}_{JL} z^L}{\text{Im } \mathcal{G}_{MN} z^M z^N} , \quad (2.6)$$

with $z^I = (i, z^a)$. In type II string theory reduced on, in particular, a Calabi–Yau manifold, ϕ and σ are identified with the four-dimensional dilaton and universal axion, respectively, while ξ^I and $\tilde{\xi}_I$ are identified with four-dimensional scalar fields arising from the RR sector.

2.2 $SU(3)$ compactifications

Let us now briefly discuss the notion of an $SU(3)$ compactification. Recall first the moduli that appear when the compactification space X_6 is a Calabi–Yau manifold. One finds that the scalar fields in vector- and hypermultiplets arise from $h^{(1,1)}$ deformations of the complexified Kähler form $J + iB$ and from $h^{(1,2)}$ deformations of the complex structure, or equivalently deformations of the holomorphic 3-form Ω . Since these moduli can be varied independently, their moduli space is locally a product

$$M(X_6) = M_J \times M_\Omega , \quad (2.7)$$

with each component being a special-Kähler manifold. Their respective Kähler potentials are given, prior to α' and string loop corrections, by [17]

$$K_J = -\ln \int_{X_6} J \wedge J \wedge J , \quad K_\Omega = -\ln i \int_{X_6} \Omega \wedge \bar{\Omega} . \quad (2.8)$$

In type IIA compactifications one has $M_J = M_v$ while $M_\Omega = M_{\text{SK}}^{2h^{(1,2)}}$ is the special-Kähler base in the c-map (2.4). The full space M_h also includes the dilaton and axion plus $2h^{(1,2)} + 2$ scalars from the RR sector which parameterize the torus fibre. In type IIB compactifications the situation is reversed and one has $M_\Omega = M_v$ while $M_J = M_{\text{SK}}^{2h^{(1,1)}}$ is the special-Kähler base. The dilaton and axion plus $2h^{(1,1)} + 2$ scalars from the RR sector again complete M_h .

In this paper we will be considering the weaker case where we only assume that X_6 admits an $SU(3)$ structure [1]. This means we can still find a globally defined fundamental 2-form J and an almost complex structure with a corresponding globally defined complex $(3,0)$ -form Ω . However, they are now no longer closed, so that

$$dJ \neq 0 , \quad d\Omega \neq 0 , \quad (2.9)$$

⁴For type IIA, we denote the coordinates and prepotential of $M_{\text{SK}}^{2n_h-2}$ in the hyper-sector by z^a and $\mathcal{G}$, respectively, in order to distinguish it from the coordinates t^i and prepotential $\mathcal{F}$ in the vector multiplet sector. For type IIB the rôles of t^i and z^a and $\mathcal{F}$ and $\mathcal{G}$ will be reversed.

and one says that the intrinsic torsion of the $SU(3)$ structure on X_6 is non-zero. This lack of integrability means that the space of generic $SU(3)$ structures on X_6 is infinite-dimensional, though it nonetheless still decomposes as a product of special-Kähler manifolds [8, 9], with Kähler potentials given by (2.8). In the corresponding ‘low-energy’ effective action the intrinsic torsion plays the role of gauge charges and/or mass terms. As a consequence, the four-dimensional supergravity is gauged.

For a realistic theory, we would like to identify a finite-dimensional subspace of $SU(3)$ structures, as we had for the Calabi–Yau case. This should correspond to identifying some light “moduli” fields in the Kaluza–Klein spectrum of all possible deformations of J and Ω . In this paper we will simply assume such an expansion exists and then investigate the consequences for quantum corrections to the corresponding effective action. Our assumptions are (see [2, 8, 9, 12] for details):

1. There is a finite-dimensional subspace M of $SU(3)$ structures such that we have the expansion

$$J + iB = t^i \omega_i, \quad \Omega = z^I \alpha_I - \mathcal{G}_I(z) \beta^I. \quad (2.10)$$

In contrast to the Calabi–Yau case, the lack of integrability (2.9) means that the basis two-forms ω_i and the dual four-forms $\tilde{\omega}^i$ as well as the three-forms α_I and β^I are no longer necessarily harmonic; these forms constitute a closed set under the Hodge star and also under the exterior derivative, namely

$$\begin{aligned} d\alpha_I &\sim e_{Ii} \tilde{\omega}^i, & d\beta^I &\sim p_i^I \tilde{\omega}^i, \\ d\omega_i &\sim p_i^I \alpha_I - e_{iI} \beta^I, & d\tilde{\omega}^i &\sim 0, \end{aligned} \quad (2.11)$$

where e_{iI} and p_i^I are constant matrices.

2. There are no additional light spin-3/2 fields other than the $\mathcal{N} = 2$ gravitini (this corresponds to an absence of moduli of type (1, 0) with respect to the almost complex structure [8, 9]).
3. There is an expansion around the large-volume limit. The moduli space manifestly includes the volume modulus V since this is a rescaling of the real parts of t^i , here we in addition assume that the moduli are light compared with other Kaluza–Klein modes in the large-volume region.

This set-up can be viewed as the $SU(3)$ structure analogue of a Calabi–Yau compactification with flux. The flux generates a potential for the Calabi–Yau moduli, but in the large volume limit this potential is generically small compared to the masses of the Kaluza–Klein modes. As a result the low-energy theory is just a gauged $\mathcal{N} = 2$ supergravity theory for the moduli fields. Here we are requiring the existence of a family of $SU(3)$ structures which are “close” to satisfying the Einstein equations, in the sense that the Ricci curvature is small compared to the Kaluza–Klein scale in the large volume limit.

The existence of an $SU(3)$ structure implies that the effective field theory in four dimensions is $\mathcal{N} = 2$ supersymmetric. Given our assumptions, the moduli space is again a product

$$M = M_J \times M_\Omega \quad (2.12)$$

of a moduli space M_J of deformations of $J + iB$ and of a moduli space M_Ω of deformations of Ω , where the volume modulus V is part of M_J . Furthermore, the expansion (2.10) is such that the special Kähler structures on the infinite dimensional space of all $SU(3)$ -structures restrict to special Kähler structures on each component M_J and M_Ω with Kähler potentials again given by (2.8). In addition, prior to including α' and string loop corrections, the low-energy effective theory is a gauged $\mathcal{N} = 2$ supergravity theory.

The RR potentials are assumed to admit an expansion in the same bases, namely α_I and β^I in type IIA and ω_i , $\tilde{\omega}^i$ and the volume form ε in type IIB, so that

$$\begin{aligned} C_3 &= \xi^I \alpha_I + \tilde{\xi}_I \beta^I , \\ C_0 &= \xi^0 , \quad C_2 = \xi^i \omega_i , \quad C_4 = \tilde{\xi}_i \tilde{\omega}^i , \quad C_6 = \tilde{\xi}_0 \varepsilon . \end{aligned} \quad (2.13)$$

In Calabi–Yau compactifications the metric on the quaternionic manifold M_h takes the Ferrara–Sabharwal form (2.5) to leading order. Related to the torus fibre the metric has $2n_h + 1$ (perturbative) Peccei–Quinn shift symmetries which read

$$\sigma \rightarrow \sigma + \gamma + c^I \tilde{\xi}_I , \quad \xi^I \rightarrow \xi^I + c^I , \quad \tilde{\xi}_I \rightarrow \tilde{\xi}_I + \tilde{c}_I , \quad \gamma, c^I, \tilde{c}_I \in \mathbb{R} . \quad (2.14)$$

These symmetries arise from large gauge transformations of the type II p -form fields C_p , that is, shifting them by constant multiples of the (closed) basis forms ω_i etc. They are broken to discrete shift symmetries by non-perturbative space-time physics but nevertheless imply a perturbative non-renormalization theorem in that perturbative corrections of the hypermultiplets only occur at one-loop but not beyond [18–20, 22, 23].

By contrast, for $SU(3)$ structures the basis forms are generically no longer harmonic, so in general the shift symmetries (2.14) are broken and the corresponding scalar fields are massive. Nonetheless, simple dimensional analysis shows that the leading-order calculation of the kinetic terms does not see the derivatives of the basis forms and so the metric on the hypermultiplet space still takes the Ferrara–Sabharwal form (2.5). At higher order in α' and g_s one would naively expect that corrections to the metric see the fact that the Peccei–Quinn symmetries are broken.

In summary, our assumptions imply that at leading order – just as in the Calabi–Yau case – in type IIA one has $M_J = M_v$ while M_Ω is the base of a special quaternionic–Kähler manifold M_h . In type IIB the situation is reversed and one has $M_\Omega = M_v$ while M_J is the base of M_h . In both cases the special quaternionic manifold M_h includes the dilaton and axion.

3 Perturbative α' and g_s corrections

In this section we discuss the structure of the perturbative α' and g_s corrections to the moduli spaces metrics in the class of $SU(3)$ structure compactifications defined above, focussing on the leading-order contributions. The key point is that they are strongly constrained by the $\mathcal{N} = 2$ supersymmetry which by construction survives the compactification. In essence we find that several of the usual arguments that apply to Calabi–Yau compactifications go through in this case too.

First note that if the low energy spectrum contains no massive spin-3/2 multiplets, $\mathcal{N} = 2$ supersymmetry enforces the split (2.1) into locally independent special-Kähler and quaternionic moduli spaces, which thus has to persist after including perturbative and non-perturbative α' and g_s corrections. The α' corrections enter in an expansion in the inverse of the volume modulus V^{-1} , while the loop corrections are in powers of the four-dimensional dilaton $e^{2\phi} = V^{-1}e^{2\varphi}$. However, in both type II compactifications the dilaton is part of a hypermultiplet and thus the component M_h receives quantum corrections, whereas M_v is uncorrected and thus “exact” already at the string tree level.⁵ The volume modulus on the other hand, resides in M_J . Therefore, in type IIA α' corrections appear in M_v whereas in type IIB they correct M_h . This situation is identical to the situation for Calabi–Yau compactifications and is summarized in Table 1.

It is important to note that for the leading order corrections there are generically mixing terms of the form $\partial_\mu \phi \partial^\mu \ln V$ between the dilaton and the volume. For Calabi–Yau compactifications, the tree-level coefficient of this term has been computed in [21, 22] but the one-loop correction has only been inferred indirectly in [22] by insisting that the split (2.1) continues to hold at the loop level. To diagonalize such terms, one requires mixing between the dilaton and volume modulus in defining the combinations that appear in the appropriate vector and hypermultiplet moduli spaces. Such mixing cannot affect the leading-order contributions to the moduli space metrics but will enter at higher order.

corrections	M_v^{IIA}	M_h^{IIA}	M_v^{IIB}	M_h^{IIB}
α'	yes	no	no	yes
g_s	no	yes	no	yes

Table 1: Structure of α' and g_s corrections in type IIA and type IIB.

In the following, we first consider only the metric moduli $\text{Re } t^i$ and z^a and use the splitting (2.1) and our assumptions about the expansion (2.10) to constrain the form of the

⁵Here *exact* is in quotation marks since at special points in the moduli space – for example at the conifold point – non-perturbative corrections which are not governed by the dilaton can occur and correct M_v [31].

leading corrections to the metrics on M_v and M_h . We then consider the dependence of the NS B -field and RR moduli, using analogues of the standard Calabi–Yau compactification arguments to investigate how first the full perturbative vector-multiplet α' corrections and g_s corrections are constrained.

3.1 Constraints on the metric moduli

Let us first focus on the leading perturbative corrections both in α' and g_s for the metric moduli $\text{Re } t^i$ and z^a . One can distinguish two types of terms. One is the contribution of pointlike massive string states, either in tree-level exchange or in one-loop diagrams. Given that we are expanding around a large volume limit of the compactification space, the masses of these states are much larger than the inverse radius of X_6 . Hence they are intrinsically ten-dimensional effects and correspond to a dimensional reduction of the ten-dimensional string tree-level and one-loop effective action. The second type of term is intrinsically four-dimensional and give “threshold effects” arising from integrating out heavy states coming from Kaluza–Klein modes and wound string states on X_6 . For type II Calabi–Yau compactifications one notably finds that all such threshold corrections in fact vanish [19].

Before going into the details of the α' and g_s corrections of the four-dimensional theory, to be discussed in the next subsections, let us recall the structure of the higher order string tree-level and one-loop terms for the gravitational modes of the ten-dimensional effective action. They have the form (see e.g. [19, 32])

$$\begin{aligned}\mathcal{L}_{\text{IIA}}|_{R^4} &\sim \zeta(3) e^{-2\varphi} (t_8 t_8 + \tfrac{1}{4} \varepsilon \varepsilon) R^4 + 2\zeta(2) (t_8 t_8 - \tfrac{1}{4} \varepsilon \varepsilon) R^4 + B \wedge I_8 , \\ \mathcal{L}_{\text{IIB}}|_{R^4} &\sim \zeta(3) e^{-2\varphi} (t_8 t_8 + \tfrac{1}{4} \varepsilon \varepsilon) R^4 + 2\zeta(2) (t_8 t_8 + \tfrac{1}{4} \varepsilon \varepsilon) R^4 ,\end{aligned}\tag{3.1}$$

where φ is the ten-dimensional dilaton, ζ is the Riemann zeta function, and

$$\begin{aligned}(t_8 t_8 \pm \tfrac{1}{4} \varepsilon \varepsilon) R^4 &\equiv (t^{M_1 \dots M_8} t_{N_1 \dots N_8} \pm \tfrac{1}{4} \varepsilon^{PQM_1 \dots M_8} \varepsilon_{PQN_1 \dots N_8}) \\ &\quad \times R_{M_1 M_2}{}^{N_1 N_2} \dots R_{M_7 M_8}{}^{N_7 N_8} .\end{aligned}\tag{3.2}$$

$R_{M_1 M_2}{}^{N_1 N_2}$ denotes the Riemann tensor, ε is the totally antisymmetric Levi–Civita tensor, and t_8 is antisymmetric in each successive pair of indices and symmetric under the exchange of any two pairs. Given an antisymmetric tensor M_{MN} , the t -tensor contraction reads

$$t_{M_1 \dots M_8} M^{M_1 M_2} \dots M^{M_7 M_8} = 24 \text{tr } M^4 - 6(\text{tr } M^2)^2 .\tag{3.3}$$

We also have

$$4608 (2\pi)^4 I_8(R) \equiv t_{N_1 \dots N_8} R^{N_1 N_2} \wedge \dots \wedge R^{N_7 N_8} = [24 \text{tr } R^4 - 6(\text{tr } R^2)^2] ,\tag{3.4}$$

where $R^{N_1 N_2}$ is the curvature 2-form. Note that the supersymmetric completion of these terms will include, in the NSNS sector, higher derivative objects built from curvatures,

$H = dB$ and derivatives of the dilaton, which would be relevant if for instance one was interested in backgrounds with non-trivial H -flux. However, although recently some significant progress has been made [33], less is known about the exact form of these terms.

3.1.1 α' corrections

Let us start by considering the leading α' corrections. As usual, all the contributions arise from the ten-dimensional effective action (3.1).⁶ In principle, this can be derived by directly performing a Kaluza–Klein reduction of (3.1). Since $t_8 t_8 R^4$ never involves contractions of indices on the same Riemann tensor, in the four-dimensional effective action any correction to the Einstein term must come from $\varepsilon \varepsilon R^4$ [19]. More precisely, the only contraction that gives a term with only two space-time derivatives is where PQ in (3.2) are four-dimensional space-time indices, and the two other space-time indices on each ε contract the same Riemann tensor. (If they contract different Riemann tensors one obtains higher-derivative scalar field terms.) Integrating over the internal space, we obtain

$$\begin{aligned} & \int_{X_6} \varepsilon^{\rho\sigma\mu_1\mu_2} \varepsilon^{m_1\cdots m_6} \varepsilon_{\rho\sigma\nu_1\nu_2} \varepsilon_{n_1\cdots n_6} R_{\mu_1\mu_2}{}^{\nu_1\nu_2} R_{m_1m_2}{}^{n_1n_2} R_{m_3n_4}{}^{n_3n_4} R_{m_5n_6}{}^{n_5n_6} \\ & \sim \mathcal{R} \int_{X_6} (\varepsilon^{m_1\cdots m_6} \varepsilon_{n_1\cdots n_6} R_{m_1m_2}{}^{n_1n_2} R_{m_3n_4}{}^{n_3n_4} R_{m_5n_6}{}^{n_5n_6}) \sim \mathcal{R} \chi(X_6) , \end{aligned} \quad (3.5)$$

where $\mathcal{R}$ is the four-dimensional Ricci scalar and $\chi(X_6)$ the Euler characteristic of X_6 . Thus we see that for $SU(3)$ structures, the correction to the Ricci scalar term comes only from $\varepsilon \varepsilon R^4$ and is proportional to χ exactly as for Calabi–Yau compactifications.⁷

For the scalar kinetic energy corrections, we note that $\varepsilon \varepsilon R^4$ necessarily has four space-time indices and thus cannot contribute. One can, however, get a correction from the $t_8 t_8 R^4$ terms [19, 21, 34]. We will denote these corrections to the metrics on the M_J and M_Ω moduli spaces as δg_J^{tree} and $\delta g_\Omega^{\text{tree}}$, respectively, and the leading order, uncorrected metrics as g_J^0 and g_Ω^0 . Inspecting the terms in (3.1), we can now write the leading α' -

⁶Note that we are expanding in terms of supersymmetry representations. Since we are off-shell these do not necessarily correspond to eigenstates of the Laplacian, and so, in contrast to the conventional case, there are potentially threshold corrections from tree-level exchange of Kaluza–Klein modes. However, by assumption the zeroth-order low-energy effective theory is a gauged supergravity for the moduli alone. This implies that the $SU(3)$ background does not source the heavy Kaluza–Klein modes, and hence such contributions are absent. For the string winding modes, conservation of winding number means they similarly cannot be sourced. By contrast, both types of heavy modes can contribute in loops.

⁷The fact that this property does not only hold for Calabi–Yau compactifications has also been observed in [16].

correction to both the IIA and IIB Lagrangians in the string frame as [22]

$$\begin{aligned}\Delta\mathcal{L}_{\text{tree}} &\sim e^{-2\phi}\left(1 + V^{-1}c\right)\mathcal{R} \\ &+ e^{-2\phi}\left(\text{Re } g_J^0 + V^{-1}\text{Re } \delta g_J^{\text{tree}}\right)_{ij} \partial_\mu v^i \partial^\mu v^j \\ &+ e^{-2\phi}\left(g_\Omega^0 + V^{-1}\delta g_\Omega^{\text{tree}}\right)_{a\bar{b}} \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} + \dots ,\end{aligned}\tag{3.6}$$

where $v^i = \text{Re } t^i$, ϕ is the four-dimensional dilaton defined as $e^{2\phi} = V^{-1}e^{2\varphi}$ and

$$c = \frac{2\zeta(3)}{(2\pi)^3} \chi(X_6) .\tag{3.7}$$

The next step is to Weyl-rescale to the Einstein frame and then expand in terms of $e^{2\phi}$ and the inverse volume V^{-1} , which parametrize loop and α' corrections, respectively.⁸ To leading order in V^{-1} , one finds

$$\begin{aligned}\Delta\mathcal{L}_{\text{tree}} &\sim \mathcal{R} + \left(\text{Re } g_J^0 + V^{-1}(\delta g_J^{\text{tree}} - c \text{Re } g_J^0)\right)_{ij} \partial_\mu v^i \partial^\mu v^j \\ &+ \left(g_\Omega^0 + V^{-1}(\delta g_\Omega^{\text{tree}} - c g_\Omega^0)\right)_{a\bar{b}} \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} + \dots .\end{aligned}\tag{3.8}$$

Recalling that there can be no α' corrections to the complex structure moduli space we immediately have

$$\delta g_\Omega^{\text{tree}} = c g_\Omega^0 .\tag{3.9}$$

One might be tempted to argue, for example by invoking a putative mirror symmetry, that $\text{Re } \delta g_J$ is similarly proportional to $\text{Re } g_J^0$, as it is in the Calabi–Yau case. In the next section, by considering the contributions of the imaginary parts of the moduli coming from the NS B -field, we will see that this is indeed the case.

3.1.2 g_s corrections

For loop corrections both ten-dimensional and threshold contributions are relevant, and – a priori – there is little one can deduce about the form of the four-dimensional low-energy action. Before Weyl-rescaling the loop-corrected Lagrangians have the form

$$\begin{aligned}\Delta\mathcal{L}_{\text{loop}}^{\text{IIA/IIB}} &\sim \left(Ve^{-2\varphi} + f^{\text{A/B}}\right)\mathcal{R} + \left(Ve^{-2\varphi} \text{Re } g_J^0 + \text{Re } \delta g_J^{\text{loop,A/B}}\right)_{ij} \partial_\mu v^i \partial^\mu v^j \\ &+ \left(Ve^{-2\varphi} g_\Omega^0 + \delta g_\Omega^{\text{loop,A/B}}\right)_{a\bar{b}} \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} + \dots ,\end{aligned}\tag{3.10}$$

where $\delta g_J^{\text{loop,A/B}}$ and $\delta g_\Omega^{\text{loop,A/B}}$ denote the leading order loop corrections of the two metrics while $f^{\text{A/B}}$ parametrize the loop-correction to the Einstein term. All three corrections

⁸Just to reiterate, the definitions of both the four-dimensional dilaton ϕ and the volume V change due to their mixing just discussed [21, 22]. However this, and any more general mixing that might appear for generic $SU(3)$ backgrounds, does not change the corrections to the moduli space metrics at this order.

contain a contribution from the reduction of the ten-dimensional terms given in (3.1) and in principle an additional threshold correction. After rescaling to the Einstein frame one finds

$$\begin{aligned} \Delta\mathcal{L}_{\text{loop}}^{\text{IIA/IIB}} \sim \mathcal{R} + & \left(\text{Re } g_J^0 + e^{2\phi} \left(\text{Re } \delta g_J^{\text{loop,A/B}} - f^{\text{A/B}} \text{Re } g_J^0 \right) \right)_{ij} \partial_\mu v^i \partial^\mu v^j \\ & + \left(g_\Omega^0 + e^{2\phi} \left(\delta g_\Omega^{\text{loop,A/B}} - f^{\text{A/B}} g_\Omega^0 \right) \right)_{a\bar{b}} \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} + \dots \end{aligned} \quad (3.11)$$

The requirement that there be no loop corrections to the vector multiplet moduli means that

$$\text{Re } \delta g_J^{\text{loop,A}} = f^{\text{A}} \text{Re } g_J^0, \quad \delta g_\Omega^{\text{loop,B}} = f^{\text{B}} g_\Omega^0. \quad (3.12)$$

Since g_J can only depend on the v^i and g_Ω can only depend on the z^a we can infer that f^{A} is a function of only v^i and f^{B} is a function of only z^a .

At this point we can put no further constraints on the corrections. However, it is interesting to note that for Calabi–Yau compactifications the type IIA and IIB corrections are related by

$$f^{\text{A}} = -f^{\text{B}}, \quad \text{Re } \delta g_J^{\text{loop,A}} = \text{Re } \delta g_J^{\text{loop,B}}, \quad \delta g_\Omega^{\text{loop,A}} = \delta g_\Omega^{\text{loop,B}}. \quad (3.13)$$

These relations arise by noting from which worldsheet spin-structure sectors the terms originate [19]. The Einstein term corrections come only from the odd-odd sector, while the metric corrections come from the even-even sector. Since t_8 is even under parity, one sees that in the ten-dimensional action (3.1), the one-loop $\epsilon\epsilon$ term is odd-odd, and changes sign between IIA and IIB, while the $t_8 t_8$ term is even-even and is the same for both IIA and IIB.

Using (3.5), we see that the loop corrections which arise from solely reducing the terms in the ten-dimensional action given in (3.1) do respect the relations (3.13) even for $SU(3)$ -structure backgrounds. Therefore, with the additional assumption that the same is true for the threshold contributions, one obtains

$$\text{Re } \delta g_J^{\text{loop,A}} = \text{Re } \delta g_J^{\text{loop,B}} = f \text{Re } g_J^0, \quad \delta g_\Omega^{\text{loop,A}} = \delta g_\Omega^{\text{loop,B}} = -f g_\Omega^0, \quad (3.14)$$

where $f = f^{\text{A}} = -f^{\text{B}}$. Furthermore, since the corrections to $\text{Re } g_J$ can only depend on the moduli $\text{Re } t^i$ and the corrections to g_Ω can only depend on the moduli z^a , we also have that

$$f = \text{constant}. \quad (3.15)$$

Note that we would have come to the same conclusions by assuming that there is a mirror symmetry between IIA and IIB sending f to $-f$. In fact, as we will see in section 3.3, we are also led to something very close to the relations (3.14) once we consider the form of the RR kinetic terms in the hypermultiplet sector, without needing the assumption about threshold corrections or mirror symmetry made here.

3.2 Perturbative α' corrections

As discussed in section 3.1.1, while there are necessarily no α' corrections to g_Ω , we could say nothing concrete about the corrections to g_J . However, we did not consider higher-derivative couplings of the NS B -field as they are not yet completely known [33]. Let us now address that issue and see how it might allow us to also constrain the perturbative corrections to g_J .

In the reduction, the light modes of B combine with the deformation of J , as in (2.10), to form complex scalar coordinates t^i on M_J . The $\mathcal{N} = 2$ prepotential $\mathcal{F}$ that determines the special-Kähler metric depends holomorphically on t^i and is given in the large volume limit – so as to match (2.8) – by

$$\mathcal{F}_0(t) = \kappa_{ijk} t^i t^j t^k , \quad (3.16)$$

with κ_{ijk} real constants and we take $X^0 = i$ and $X^i = t^i$. (In Calabi–Yau compactifications the κ_{ijk} are the classical intersection numbers; more generally they are related to the basis forms ω_i [8, 9].) In order to determine or constrain perturbative α' corrections, we have to determine the sub-leading corrections to $\mathcal{F}$. Expanding in large t^i we have generically

$$\mathcal{F}(t) = \mathcal{F}_0(t) + \alpha_{ij} t^i t^j + \beta_i t^i + \gamma + \hat{\mathcal{F}}(t) , \quad (3.17)$$

where $\hat{\mathcal{F}}$ contains non-perturbative corrections (i.e. instanton corrections) together with possibly negative powers of t^i . Here we use the fact that each power in the α' expansion comes with a volume factor $V^{-1/3}$. From (2.8) we see that V is cubic in t^i

$$V = \frac{1}{6} \kappa_{ijk} (t + \bar{t})^i (t + \bar{t})^j (t + \bar{t})^k = \frac{1}{6} e^{-K_0} , \quad (3.18)$$

where K_0 is the leading order Kähler potential computed from $\mathcal{F}_0$, and hence the corrections to $\mathcal{F}_0$ are in descending powers of t^i .

Inserting (3.17) into (2.3) one obtains

$$e^{-K} = \kappa_{ijk} (t + \bar{t})^i (t + \bar{t})^j (t + \bar{t})^k + a_{ij} t^i \bar{t}^j + i b_i (t - \bar{t})^i + c + \dots , \quad (3.19)$$

where

$$a_{ij} = 2i(\alpha_{ij} - \bar{\alpha}_{ij}) , \quad i b_i = (\beta_i - \bar{\beta}_i) , \quad c = 2i(\gamma - \bar{\gamma}) . \quad (3.20)$$

So we see that the real parts of α_{ij} , β_i and γ actually do not enter the Kähler potential and for our purpose may be set to zero without loss of generality.⁹ Computing the metric from (3.19) we find

$$g_{J i \bar{j}} = -e^K (6\kappa_{ij} + a_{ij}) + e^{2K} (3\kappa_i + a_{ik} \bar{t}^k + i b_i) (3\kappa_j + a_{jk} t^k - i b_j) , \quad (3.21)$$

⁹In Calabi–Yau compactifications they are determined to be non-vanishing using mirror symmetry, but this plays no role in the following.

where we abbreviated

$$\kappa_{ij} = \kappa_{ijk}(t + \bar{t})^k, \quad \kappa_i = \kappa_{ilk}(t + \bar{t})^l(t + \bar{t})^k. \quad (3.22)$$

We would like to determine the values for a_{ij} , b_i and c . Recall that for Calabi–Yau compactifications there is a perturbative Peccei–Quinn symmetry $B \rightarrow B + \lambda^i \omega_i$, which implies that g_{ij}^I has an isometry $t^i \rightarrow t^i + i\lambda^i$ for constant real λ^i . This is very constraining, implying in particular that $a_{ij} = b_i = 0$. In fact, the PQ symmetry together with the large volume limit is enough to also prove that no negative powers of t^i can appear in $\mathcal{F}$ and that the perturbative prepotential can only be $\mathcal{F}(t) = \mathcal{F}_0(t) + \gamma$ with γ constant.¹⁰

Now let us turn to the general $SU(3)$ structure case. In general the PQ symmetry is a priori not present since the basis forms are not necessarily closed. However, as discussed in the previous sub-section, the leading string corrections to supergravity, including those involving B , appear at order α'^3 . Thus all leading corrections to g_{ij}^I are suppressed by a factor of V^{-1} and hence by cubic powers in t^i . This means, simply by counting powers of t , that we expect $a_{ij} = b_i = 0$ and in fact, somewhat surprisingly, the leading order corrected metric still has the PQ symmetry.

In the analysis so far we made an implicit assumption since the expansion (3.17) in $V^{-1/3}$ strictly defines $\mathcal{F}$ as a sum of functions homogeneous under the rescaling $t^i \rightarrow \mu t^i$, that is

$$\mathcal{F}(t) = \mathcal{F}_0(t) + \mathcal{F}_1(t) + \mathcal{F}_2(t) + \cdots + \mathcal{F}_{np}(t), \quad (3.23)$$

where $\mathcal{F}_i(t)$ scales as μ^{3-i} and $\mathcal{F}_{np}(t)$ is the non-perturbative correction. Thus in (3.17) we are really extracting only the polynomial parts of each $\mathcal{F}_i(t)$ in the expansion, relegating the non-polynomial contributions to $\hat{\mathcal{F}}$. The latter would signal power-like singularities in the variables t^i and naively one might expect their absence also in the $SU(3)$ structure case, although notably, under T-duality, the contributions of wound strings are precisely of this type. For the leading α' corrections we can actually address this question directly. First, as for the α and β terms above, since the leading string corrections to supergravity, including those involving the B -field, appear at order α'^3 , we can immediately argue that $\mathcal{F}_1$ and $\mathcal{F}_2$ vanish, since they correspond to α' and α'^2 corrections. Turning to the α'^3 correction, $\mathcal{F}_3$ we have already noted that the corrections to the kinetic terms are unaffected by moduli mixing to this order. Thus we can take the zeroth-order definitions, and identify $\text{Im } t^i$ with the expansion of the B -field in (2.10). Next we note, just by power counting, that the leading higher-derivative corrections to the ten-dimensional effective action can include only finite powers of $H = dB$ up to H^4 . Hence we can only have *polynomial* dependence on $\text{Im } t^i$ in the correction to the metric δg_J . Together with the homogeneity condition this is enough to argue that in fact $\mathcal{F}_3$ is constant.

¹⁰To see this one inserts an arbitrary function into (2.3), computes the metric and imposes the PQ symmetry – this determines $\mathcal{F}(t) = \mathcal{F}_0(t) + \gamma$ (modulo terms that do not contribute to K). In particular, the term $\hat{\mathcal{F}}$ in (3.17) cannot have negative powers of t^i and only contains non-perturbative corrections which break the PQ symmetry to a discrete subgroup allowing for the dependence $\hat{\mathcal{F}}(e^{2\pi t})$.

To summarize, for any $SU(3)$ structure compactifications we have argued that the α' -corrected prepotential on M_J has the form

$$\mathcal{F} = \kappa_{ijk} t^i t^j t^k - \frac{1}{2} i \tilde{c} \quad (3.24)$$

where $\tilde{c}$ is constant. Note that this is exactly the same form as for Calabi–Yau compactifications, in which case $\tilde{c} = 2\zeta(3)\chi(X_6)/(2\pi)^3$ is proportional to the Euler characteristic (and equal to c in (3.7)). The corresponding metric is given by

$$g_{Ji\bar{j}} = -6e^K \kappa_{ij} + 9e^{2K} \kappa_i \kappa_{\bar{j}} , \quad \text{with} \quad e^{-K} = \kappa_{ijk} (t + \bar{t})^i (t + \bar{t})^j (t + \bar{t})^k + \tilde{c} , \quad (3.25)$$

so that expanding to first order in V^{-1} gives

$$g_{Ji\bar{j}} = g_{Ji\bar{j}}^0 + e^{K_0} \delta g_{Ji\bar{j}} \quad (3.26)$$

where $g_{Ji\bar{j}}^0$ and K_0 are the leading order metric and Kähler potential computed from the cubic prepotential $\mathcal{F}_0$ given in (3.16) and¹¹

$$\delta g_{Ji\bar{j}} = -\tilde{c} g_{Ji\bar{j}}^0 - 9\tilde{c} e^{2K_0} \kappa_i \kappa_{\bar{j}} . \quad (3.27)$$

This result might seem somewhat surprising as we just argued that the PQ isometries are not present in $SU(3)$ structure compactifications, yet the metric given in (3.25) does have them. This is arising because the ten-dimensional corrections have a very particular universal form, which together with $\mathcal{N} = 2$ supersymmetry imply the absence of subleading polynomial corrections in the prepotential. In that sense the PQ isometries can be viewed as “accidental” symmetries. (Of course they are broken in the potential already at leading order [8].)

The arguments used so far did not determine the value of $\tilde{c}$ for $SU(3)$ structure compactifications. It is tempting to conjecture that it is again equal to c in (3.7) and hence given by the Euler characteristic. This would be consistent with any putative mirror symmetry given the relation (3.9). It is also interesting to note that it is consistent with the arguments of [35], where it was shown that manifolds X_6 with vanishing Euler characteristic necessarily have an additional $SU(2)$ structure and the compactification can be viewed as a spontaneously broken $N = 4$ theory. The $N = 4$ supersymmetry then forbids any perturbative corrections and implies $\tilde{c} \sim \chi(X_6)$ or in other words that $\tilde{c}$ must vanish as the Euler characteristic goes to zero.¹²

¹¹Note that the second term appears to violate the conjecture in section 3.1 that the contributions to δg_J^{tree} and $\delta g_\Omega^{\text{tree}}$ from the $t_8 t_8 R^4$ term are proportional to g_J^0 and g_Ω^0 respectively. However just as in the Calabi–Yau case [21, 22], the conjecture is really that these are the forms of the contributions up to pieces arising from a total derivative term in the expansion of $t_8 t_8 R^4$. The point is that integrating such a term by parts against the $V^{-1} e^{-2\phi}$ coefficient in (3.6) generates the second term in (3.27) for δg_J but no such term for δg_Ω . The corresponding term is also absent for δg_J^{loop} in the reduction of the ten-dimensional loop correction, since there the coefficient is independent of V .

¹²We thank H. Triendl for pointing this out.

One might also wonder if, as in the Calabi–Yau case, this leading contribution actually gives the perturbative α' correction to all orders. If the argument that $\mathcal{F}_3$ is polynomial in $\text{Im } t^i$ can be extended to all $\mathcal{F}_n$ then homogeneity implies that all the higher $\mathcal{F}_n$ with $n > 3$ do indeed vanish. However, although one can again argue that the corresponding higher-derivative corrections must be polynomial in H , the fact that there may be moduli mixing means that we cannot conclude that $\mathcal{F}_n$ is generally polynomial in $\text{Im } t^i$. Thus, as stands, we cannot argue against higher-order corrections.

3.3 Perturbative g_s corrections

As we discussed above, for Calabi–Yau compactifications the zero-modes $\xi, \tilde{\xi}$ of the RR potentials C_p together with the axion σ in the hypermultiplet moduli space admit $2n_h + 1$ (perturbative) Peccei–Quinn shift symmetries given in (2.14). In $SU(3)$ compactifications the situation is more involved since generically some of the scalar fields become massive, or in other words the number of the zero modes is reduced, because the basis forms are no longer harmonic. As discussed in [2, 8, 9, 12], a subset of the shift symmetries (2.14) turns local in that $c^I, \tilde{c}_I$ become space-time dependent and appropriate couplings to the gauge fields are induced. $\mathcal{N} = 2$ supersymmetry in turn demands a non-trivial scalar potential which lifts some of the flat directions corresponding to the “non-zero” but light modes mentioned above. An argument along the lines of ref. [36] further shows that these gauged isometries survive after including perturbative and non-perturbative corrections.

The number of gauged isometries depends on the specific structure of the non-trivial torsion, that is, the constants e_{iI} and p_i^I in (2.11). Nevertheless, one can determine that for any $SU(3)$ compactification at least n_h of the isometries in (2.14) survive perturbatively, and only the question which are gauged depends on the details of X_6 [9]. To review this argument, note that in this case, for the RR scalars in (2.13),

$$\begin{aligned} d(\xi^I \alpha_I + \tilde{\xi}_I \beta^I) &= (d\xi^I) \alpha_I + (d\tilde{\xi}_I) \beta^I + (\xi^I e_{iI} + \tilde{\xi}_I p_i^I) \tilde{\omega}^i, \\ d(\xi^0 + \xi^i \omega_i + \tilde{\xi}_i \tilde{\omega}^i + \tilde{\xi}_0 \varepsilon) &= d\xi^0 + (d\xi^i) \omega_i + (d\tilde{\xi}_i) \tilde{\omega}^i + (d\tilde{\xi}_0) \varepsilon + (\xi^i e_{iI}) \alpha^I - (\xi^i p_i^I) \beta_I. \end{aligned} \quad (3.28)$$

This means that the RR field strengths depend explicitly on the combinations $\xi^I e_{iI} + \tilde{\xi}_I p_i^I$ in type IIA and $\xi^i e_{iI}$ and $\xi^i p_i^I$ in type IIB. Thus for IIB it is clear that at most one loses the $n_h - 1$ isometries $\xi^i \rightarrow \xi^i + c^i$. For type IIA, one notes that $d^2 = 0$ implies $e_{iI} p_j^I - p_i^I e_{jI} = 0$ and hence the vectors $Z_i = (e_{iI}, p_i^I)$ span an isotropic subspace of the $2n_h$ -dimensional symplectic space spanned by α_I and β^I . Thus there can be at most n_h linearly independent Z_i and hence at most n_h combinations $\xi^I e_{iI} + \tilde{\xi}_I p_i^I$ that appear explicitly in the RR field strengths and hence have broken PQ symmetry. This result implies that all hypermultiplets can be dualized to tensor multiplets, where the scalars which transform as in (2.14) are replaced by dual antisymmetric tensors [37].¹³

¹³In type IIB the tensor multiplets already arise in the field basis which naturally occurs in the Kaluza–

This property was used in [23] to parameterize the possible perturbative corrections in Calabi–Yau compactifications in terms of one scalar function $\Delta(z)$. $\mathcal{N} = 2$ supersymmetry alone already constrains the string loop-corrected scalar field spaces to be quaternionic-Kähler. Our assumption of $SU(3)$ structure further implies that M_h is a torus fibration over a special-Kähler manifold. However, because not all the RR isometries survive, the metric on M_h no longer has to be ‘special quaternionic-Kähler’, or in other words the loop-corrected metric is generically not of the Ferrara–Sabharwal form given in (2.5) with merely a loop-corrected prepotential $\mathcal{G}$. However, the existence of at least $n_h + 1$ unbroken translational isometries additionally constrains the form of the metric [18, 20, 23], and this is best described in terms of the dual tensor multiplets. For Calabi–Yau compactifications – and as we just argued, also for $SU(3)$ structure compactifications – all hypermultiplets can be dualized to tensor multiplets, and as a consequence the constraints determined in [37, 39] apply. Using the additional property that the dilaton organizes the g_s expansion, we can repeat the analysis performed in [23] and arrive at the same result that the corrections to M_h are determined by a single function Δ , that is the imaginary part of a holomorphic function of the base-coordinates z and is homogeneous of degree zero. Explicitly, the correction to (2.5) has to be of the form [23]

$$\begin{aligned} \mathcal{L} = & \frac{1 + 2\Delta e^{2\phi}}{1 + \Delta e^{2\phi}} (\partial_\mu \phi)^2 + \frac{1 + \Delta e^{2\phi}}{1 + 2\Delta e^{2\phi}} e^{4\phi} (D_\mu \sigma - \xi^I D_\mu \tilde{\xi}_I - \Delta \mathcal{A}_\mu)^2 \\ & + (1 + \Delta e^{2\phi}) g_{a\bar{b}} \partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} - \frac{e^{4\phi}}{1 + \Delta e^{2\phi}} \tilde{\mathcal{A}}_\mu \tilde{\mathcal{A}}^\mu + \Delta Y_{IJ} D_\mu \xi^I D^\mu \xi^J \\ & - e^{2\phi} \mathcal{T}^{IJ} (D_\mu \tilde{\xi}_I - 2i \mathcal{M}_{IK} D_\mu \xi^K) (D^\mu \tilde{\xi}_J + 2i \bar{\mathcal{M}}_{JL} D^\mu \xi^L) , \end{aligned} \quad (3.29)$$

where we included appropriate covariant derivatives. $\mathcal{A}_\mu = -\text{Im}(\partial_a \mathcal{K} \partial_\mu z^a)$ is the Kähler connection on the special-Kähler base, $\tilde{\mathcal{A}}_\mu = \text{Re}(\partial_a \Delta \partial_\mu z^a)$, and $\mathcal{M}_{IJ}(z, \bar{z}, \phi)$, $\mathcal{T}_{IJ}(z, \bar{z}, \phi)$ are quantum deformations of $\mathcal{N}_{IJ}$ (defined in (2.6)) and $2 \text{Re} \mathcal{N}_{IJ}$, respectively. All these corrections are proportional to powers of Δ and the precise expressions of $\mathcal{M}_{IJ}(z, \bar{z}, \phi)$, $\mathcal{T}_{IJ}(z, \bar{z}, \phi)$ and the matrix $Y_{IJ}(z, \bar{z}, \phi)$ can be found in [23]. Here we have written z^a for the coordinates on the base, corresponding to type IIA. In type IIB these would be replaced by t^i .

Again, we can compare this form of the metric directly with the loop corrections to M_J and M_Ω we discussed in section 3.1.2. It provides strong constraints on their form. Comparing (3.11) with the leading-order kinetic terms for z^a in IIA and for t^i in IIB in (3.29), we see that

$$\delta g_{\Omega}^{\text{loop,A}} = (\Delta^A + f^A) g_{\Omega}^0 , \quad \delta g_J^{\text{loop,B}} = (\Delta^B + f^B) g_J^0 , \quad (3.30)$$

where Δ^A and Δ^B are the relevant functions of z^a and t^i respectively. Again, we see that the corrections are in fact proportional to the zeroth-order metrics, consistent with the

Klein reduction [38].

conjecture (3.14). Furthermore, given the relations (3.12) and the fact that the corrections to g_Ω and g_J can only depend on z^a and t^i respectively, we can also conclude that

$$f^A = \text{constant} , \quad f^B = \text{constant} . \quad (3.31)$$

This implies that in both type IIA and IIB the renormalization of the Einstein term (including the threshold corrections) is simply given by a constant.

Also in the hypermultiplet sector we see accidental PQ symmetries. Although the expansion is not generically compatible with preserving all $2n_h + 1$ PQ symmetries, this is not realized in the correction to the hypermultiplet moduli space – the corrected metric (3.29) still preserves all the shift symmetries. Instead, the breaking is only realized in the mass terms. Furthermore, it was argued in [23] that the tensor multiplet structure and the dilaton expansion was enough to exclude any further corrections beyond one-loop. Using the same logic here, the implication is that for $SU(3)$ structure compactifications there is a non-renormalization theorem stating that the hypermultiplet metric can only be corrected perturbatively at one-loop but not beyond.

As stands we are still left with arbitrary functions $\Delta^A(z)$ and $\Delta^B(t)$. However, as discussed in section 3.1, a putative mirror symmetry points to the conjecture that the correction terms respect the same spin-structure symmetries as in Calabi–Yau compactifications and both Δ^A and Δ^B are in fact constant. If in addition we invoke the arguments of [35] we can further constrain the constant to be proportional to the Euler characteristic $\chi(X_6)$. We therefore close with the conjecture

$$\Delta^A = -\Delta^B = \text{constant} \sim \chi(X_6) . \quad (3.32)$$

4 Summary and Outlook

We have constrained the leading perturbative α' and g_s corrections to the moduli space metric arising in compactifications of type II theories on six-dimensional $SU(3)$ structure manifolds X_6 , subject to some simple assumptions about the low-energy modes. We have shown that both the α' and g_s leading corrections to the four-dimensional curvature scalar are constant, with the tree level contribution given by the Euler characteristic $\chi(X_6)$. Furthermore, the leading tree-level correction of the Kähler moduli space metric corresponds to a constant term $\tilde{c}$ in the prepotential $\mathcal{F}$ (cf. (3.24)). For the loop corrections we argued that a non-renormalization theorem holds in that the metric in the hypermultiplet sector is corrected at one-loop but receives no further perturbative correction. For the one-loop corrections we could not show that they coincide for type IIA and type IIB but they might differ by a moduli dependent function $\Delta^{A/B}$ (cf. (3.12), (3.30)). We summarize these results in Table 2.¹⁴

¹⁴Note that here we are presenting these corrections after Weyl-rescaling to the Einstein frame, in contrast to the expressions in section 3.1. Also, the full tree-level correction to δg_J is given by (3.27). For simplicity, we use a shorthand of only writing the first term in (3.27) (see comment in footnote 11).

	δg_J		δg_Ω	
	tree	loop	tree	loop
IIA	$-\tilde{c}g_J^0$	0	0	$\Delta^A g_\Omega^0$
IIB	$-\tilde{c}g_J^0$	$\Delta^B g_J^0$	0	0

Table 2: Summary of metric corrections.

While our analysis leaves $\tilde{c}$ and $\Delta^{A/B}$ undetermined, we were able to gather a variety of arguments which point to the fact that they are exactly as in Calabi-Yau compactifications, namely that $\Delta^{A/B}$ is constant and all constants are proportional to the Euler characteristic

$$\tilde{c} \sim \Delta^{A/B} \sim \chi(X_6) . \quad (4.1)$$

This would also imply that, even though the mass terms originating from the gaugings break some of the PQ shift symmetries, the perturbatively corrected kinetic terms actually possess *all* shift symmetries. In fact, the derivation of (3.29) in [23] shows that this property holds even for non-constant functions $\Delta^{A/B}$ under some assumptions on their pole structure.

A key open question is how dependent these results are on our assumptions about the low-energy modes, including the absence of light spin-3/2 particles. We hope nonetheless that this work provides a useful basis point for studying the form of general corrections to non-Calabi-Yau compactifications. We also have only considered the case of $SU(3)$ structure manifolds, without background fluxes. An obvious extension of our results would be to consider more general $SU(3) \times SU(3)$ -structure manifolds, and/or NSNS and RR fluxes. One could also try to investigate the form of the non-perturbative corrections. We hope to make progress on these issues in the near future.

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