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Universal Entanglement Crossover of Coupled Quantum Wires

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We consider the entanglement between two one-dimensional quantum wires (Luttinger liquids) coupled by tunneling through a quantum impurity. The physics of the system involves a crossover between weak and strong coupling regimes characterized by an energy scale T_B , and methods of conformal field theory therefore cannot be applied. The evolution of the entanglement in this crossover has led to many numerical studies, but has remained little understood, analytically or even qualitatively. We argue in this Letter that the correct universal scaling form of the entanglement entropy S (for an arbitrary interval of length L containing the impurity) is $\partial S / \partial \ln L = f(LT_B)$. In the special case where the coupling to the impurity can be reffermionized, we show how the universal function $f(LT_B)$ can be obtained analytically using recent results on form factors of twist fields and a defect massless-scattering formalism. Our results are carefully checked against numerical simulations.

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Introduction.—The study of two one-dimensional gapless systems connected by some sort of interaction has become paradigmatic in modern quantum physics. It plays a particularly important role in the context of local quenches, transport through quantum dots, and the dynamics of magnetic impurities.

An essential feature of these systems is the existence of crossover scales, which play a role similar to the Kondo temperature in the Kondo problem [1], and qualitatively separate weak and strong coupling regimes. These scales make the methods of conformal field theory inapplicable, and exact results are very scarce. At the same time, the presence of the crossover indicates very rich physics.

A case in point is the so-called Kane-Fisher problem [2], where a single impurity in a Luttinger liquid has the dramatic effect of decoupling the two sides (repulsive case) or disappearing (attractive case) as energy is swept across the crossover scale T_B . This problem appears in various other guises, in particular in experiments where a fractional quantum Hall fluid is pinched by a gate voltage [3,4]. A quantity of crucial interest is then the entanglement entropy S of a region (of length L) bounded by the impurity with the rest of the system. In the case where the impurity (defect) is marginal [5], or for some classes of strongly disordered critical points [6], one can argue that $S \propto \ln L$. However, when the impurity is characterized by a crossover scale T_B , general arguments show that S has a logarithmic behavior only in the low and high energy limits, with different prefactors. The question of how S interpolates between these—both qualitatively and quantitatively—has remained largely open to now.

An early study [7] attempted a perturbative approach, with results in disagreement with numerics [8]. The problem was revisited several times (see, e.g., Ref. [9] for a review) before it was realized that, in fact, the entanglement in this problem is nonperturbative (at $T = 0$) [10,11]. Similar questions arise in the—maybe even more interesting physically—case where the tunneling between the Luttinger liquids takes place through a resonant level (quantum dot) [16].

Unfortunately, nonperturbative approaches are few, especially for the entanglement, which is essentially a nonlocal quantity. Even when problems are in appearance “free,” and involve a quadratic fermionic Hamiltonian, the nonlocality of S makes analytical calculations difficult, much like those involving observables that are nonlocal in terms of the fermions (e.g., the spin in the Ising model). We report in this Letter the solution of this problem in such a “free” fermionic case, which we obtain by the combination of a massless form-factors approach and a factorized scattering description that involves both reflection and transmission channels. We obtain results over the whole crossover, which are extremely well matched by numerical simulations. We also give the scaling form of the entanglement, which we argue generalizes to interacting situations.

XXZ spin chains and impurities.—We consider two semi-infinite spin- $\frac{1}{2}$ XXZ spin chains (spinless interacting fermions) in the gapless Luttinger liquid (LL) phase (with anisotropy $-1 < \Delta \leq 1$) connected through either a weak link, or a quantum dot (two successive weak links). The Hamiltonian is

$$H = \sum_{i=-\infty}^{-2} \vec{S}_i \cdot \vec{S}_{i+1} + \sum_{i=1}^{\infty} \vec{S}_i \cdot \vec{S}_{i+1} + H_{\text{imp}}, \quad (1)$$

where $\vec{S}_i \cdot \vec{S}_{i+1} = S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z$ is shorthand for the anisotropic interaction. The tunneling between the two interacting wires is described by $H_{\text{imp}}^{\text{wl}} = J' S_{-1}^+ S_1^- + \text{H.c.}$ in the weak link case, or $H_{\text{imp}}^{\text{dot}} = J'(S_{-1}^+ + S_1^+) S_0^- + \text{H.c.}$ in the dot case, with $S^\pm = S_x \pm iS_y$. We work at zero temperature so that the system is in a pure state.

Entanglement entropy and mutual information.—To characterize the entanglement between the two wires, we consider two geometries (Fig. 1). We are mostly interested in the entanglement entropy (EE) $S(\ell, L)$ of an interval $[-L + \ell, \ell]$ of length L not necessarily centered on the impurity [see Fig. 1(a)]. We characterize this asymmetry by the parameter $\alpha = \ell/L$. Recall that the EE can be computed as $S = -\text{tr} \rho \ln \rho$, where ρ is the reduced density matrix obtained by tracing over the degrees of freedom outside the considered interval. The symmetric case $\alpha = \frac{1}{2}$, which has been studied extensively recently [9–15], is natural for the Kondo problem, since the impurity lies at the boundary of a half-infinite chain in a folded picture [16]. However, the limit $\alpha \rightarrow 0$ is more meaningful for two wires connected by an impurity, since it provides information on their entanglement. Another natural quantity is the mutual information (MI) $I(L)$ of two intervals of length L : $A = [-L, 0]$ and $B = [0, L]$ [see Fig. 1(b)]. The MI of A and B , defined by $I = S_A + S_B - S_{A \cup B}$, is positive and relates to the EE through $I(L) = 2S(\ell = 0, L) - S(\ell = L, 2L)$. The MI characterizes the correlations between two intervals, and provides an upper bound on their entanglement. In our problem, the MI vanishes when the two wires are decoupled ($J' = 0$). A full characterization of the entanglement between the wires would require more precise estimators such as the negativity [17] (see, e.g., Refs. [18–20] in the context of the Kondo problem), for which we expect the scaling predictions of this Letter to hold as well. We emphasize that the limit $\alpha \rightarrow 0$ of the EE is

crucial when computing the MI, as it contains information on the entanglement between the two wires.

Bosonization and RG analysis.—We study the entanglement in the physically interesting case where the impurity is relevant in the renormalization group (RG) sense. We consider energies much smaller than the bandwidth, where field-theoretic results are applicable. The large-distance physics of the two XXZ half chains can then be described by a LL with Luttinger parameter $g^{-1} = 2 - (2/\pi) \arccos \Delta$ [21]. After bosonization, the LL theory consists of a massless compactified boson, with right and left moving components scattering on the impurity. Unfolding the semi-infinite wires to obtain chiral bosons on the real line, one finds

$$H = \frac{v}{2\pi} \int \sum_{a=1,2} dx (\partial_x \phi_a)^2 + H_{\text{imp}}[\phi_a(0)], \quad (2)$$

where $a = 1, 2$ labels the wires. The impurity interaction reads $H_{\text{imp}}^{\text{wl}} = \lambda \cos \sqrt{2/g}(\phi_2(0) - \phi_1(0))$ (respectively, $H_{\text{imp}}^{\text{dot}} = \lambda S_0^+ \sum_a e^{-i\sqrt{2/g}\phi_a(0)} + \text{H.c.}$) in the weak link (respectively, dot) case with $\lambda \propto J' + \dots$. Therefore [22], the weak link impurity has dimension g^{-1} and is relevant for attractive interactions ($\Delta < 0$, $g > 1$) only. The system “heals” under renormalization, flowing to a strong-coupling fixed point where the impurity is fully hybridized with the wires. The crossover is characterized by the energy scale $T_B \propto (J')^{1/(1-g)}$. Conversely, the dot impurity is always relevant, and at strong coupling the impurity is screened over a typical length scale $\xi_B \sim T_B^{-1}$ (the “Kondo screening cloud”), with $T_B \propto (J')^{2/(2-g)}$.

Other impurity problems can be treated similarly, including the anisotropic Kondo problem, the interacting resonant level model, or the tunneling between fractional quantum Hall edges [3,4,23]. The resulting chiral field theory can be folded back into an integrable boundary problem, which is usually convenient to perform calculations. In our case, however, we stress that folding procedures are incompatible with the asymmetric geometry of Fig. 1(a), and one must maintain the original unfolded formulation, which is non-integrable in general.

Ultraviolet (UV) and infrared (IR) limits, perturbation theory.—The difficulty of computing the EE $S(\ell, L)$ in this impurity problem stems from the energy scale T_B , and the asymmetric geometry. The weak and strong coupling limits can, however, easily be analyzed using conformal field theory (CFT) results [24,25]. In the weak coupling (UV) limit ($L, \ell \ll T_B^{-1}$) the physics is essentially given by two decoupled wires with the interval at one boundary, so the EE reads

$$S_{\text{UV}} \sim \frac{1}{6} \left[\ln \frac{L - \ell}{a} + \ln \frac{\ell}{a} \right], \quad (3)$$

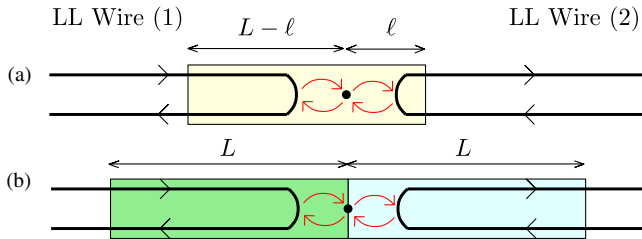


FIG. 1 (color online). Geometries considered in this Letter. We consider two Luttinger liquids (LLs) connected through an impurity, here a quantum dot. We are interested in (a) the entanglement entropy $S(\ell, L)$ of an interval, not necessarily centered on the impurity ($\ell \neq L/2$), and in particular in the limit $\ell \ll L$, and (b) the mutual information $I(L)$ of two intervals of size L , with the impurity at their extremity.

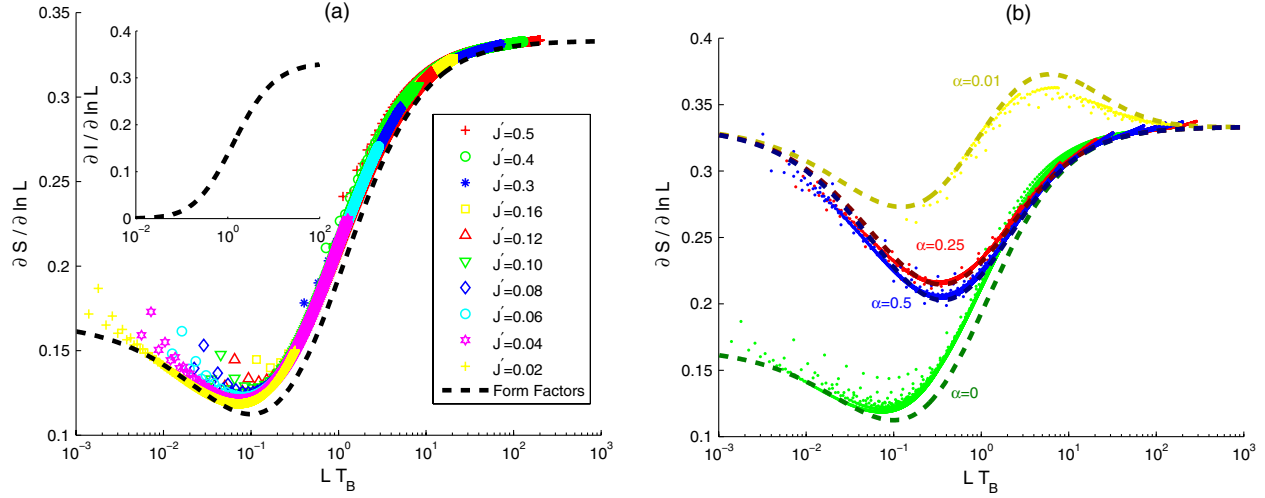


FIG. 2 (color online). EE scaling function $f(x, \alpha)$ with $x = LT_B$ and $\alpha = \ell/L$. The FF approximation (8), shown as dashed lines, is compared with numerics for two wires of each $N = 32\,000$ sites and several values of J' . (a) The limit $\alpha \rightarrow 0$. The inset shows the MI, without numerics. (b) Different values of α . For clarity, the numerical data for all values of J' here carry the same color.

where a is a UV cutoff (lattice spacing), and we have inserted the central charge $c = 1$ of the LL liquid. For $\alpha = (\ell/L) \neq 0$, this becomes $S_{UV} \sim 2 \times \frac{1}{6} \ln L$ for large L , whereas for $\alpha \rightarrow 0$ (i.e., $\ell \sim a$), one has instead $S_{UV} \sim \frac{1}{6} \ln L$, since the interval contains only a single half wire. Clearly, S_{UV} contains nonuniversal terms when $\alpha \rightarrow 0$, since the limiting procedure necessarily refers to the lattice spacing.

In the strong coupling (IR) limit ($L, \ell \gg T_B^{-1}$) the interval is in the bulk of a single healed wire, whence

$$S_{IR} \sim \frac{1}{3} \ln \frac{L}{a}. \quad (4)$$

Thus, for $\alpha \rightarrow 0$, the logarithmic term of the EE increases under renormalization: $\frac{1}{6} \ln L \rightarrow \frac{1}{3} \ln L$. This increase is expected, and witnesses to the “healing” of the chain upon renormalization.

Concerning the MI, one has $I_{UV} = 0$ at high energy, since the chains are decoupled, and $I_{IR} \sim \frac{1}{3} \ln L$ at low energy. Starting from these well-understood fixed points at tunneling amplitudes $\lambda = 0$ and $\lambda = \infty$, one could hope to compute the EE or the MI perturbatively. However, the conclusions of Ref. [10], obtained for a symmetric interval, apply to any α . The weak-coupling expansion of $S(\ell, L)$ is plagued by strong infrared divergences, indicating a non-analytic behavior in λ , while the strong-coupling expansion can, in principle, be computed following Refs. [11,12], although it would fail to capture the crossover physics. The nonperturbative nature of the weak-coupling expansion has unfortunately been overlooked previously.

Universal scaling of the entanglement.—Even though $S(\ell, L)$ (including the limit $\ell/L \rightarrow 0$) cannot be computed in general, one can still infer its universal scaling form. For a symmetric interval ($\alpha = \frac{1}{2}$), it was argued [12] that

$S(\ell = L/2, L) - S_{IR}$ is a universal function of LT_B . Also in our case, we expect the EE to be related to a universal scaling function, interpolating between the weak and strong coupling regimes. However, it is clear from the evolution of the $\ln L$ term under the RG flow discussed above that $S(\ell, L) - S_{IR}$ itself cannot be a scaling function of LT_B for all values of ℓ/L . Instead, we shall argue that the EE admits a general scaling

$$\frac{\partial S(\alpha = \ell/L, L)}{\partial \ln L} = f(LT_B, \ell/L) \quad (5)$$

with $f(0, 0) = 1/6$ and $f(0, \ell/L \neq 0) = 1/3$ in the UV limit, and $f(\infty, \ell/L) = 1/3$ at low energy. This scaling formula is physically appealing as it somehow follows the $\ln L$ term during the flow. Consequently $f(LT_B, \ell/L)$ can be thought of as some kind of “effective central charge,” thus allowing a more precise interpretation of the numerics in Ref. [8], where a “length-dependent effective central charge” was introduced. One must be careful, however, since the derivative with respect to $\ln L$ obviously picks up other terms that are not logarithmic in L .

Our main result (5) can be obtained from the scaling of the Renyi entropy $S_n = (1/1 - n) \ln R_n$, with $R_n = \text{tr} \rho^n$ and ρ the reduced density matrix introduced above. Recall that the EE can be computed from a replica trick as $S = -(d/dn) R_n|_{n=1}$. The crucial point is the identification of R_n as a two-point function of twist fields on an n -sheeted Riemann surface [24]. In our context of a $c = 1$ CFT with a relevant boundary perturbation, we expect R_n to scale as (dropping the ℓ dependence for simplicity)

$$R_n = \text{tr} \rho^n = c_n \left(\frac{L}{a} \right)^{-\frac{1}{6}(n-n^{-1})} \Omega(LT_B, n), \quad (6)$$

with $c_1\Omega(LT_B, n=1) = 1$ so that $R_1 = 1$. Here we have separated the universal scaling function $\Omega(LT_B, n)$ coming from the two-point function, and the nonuniversal proportionality coefficients c_n that can be thought of as functions of aT_B —they can evolve during the flow, and they depend explicitly on the UV cutoff a . The entanglement entropy can thus be expressed as $S = h(LT_B) + k(aT_B)$, where $h(LT_B) = -\partial_n \ln \Omega|_{n=1} + \frac{1}{3} \ln LT_B$ and $k(aT_B) = -\partial_n \ln c_n|_{n=1} - \frac{1}{3} \ln aT_B$. To get rid of the nonuniversal contribution in the general case, we consider a derivative with respect to $\ln L$ to find Eq. (5) as claimed.

Free Fermions exact solution.—At the free fermion point $\Delta = 0$, the scaling function (5) can be computed exactly by combining the form factor (FF) approach of Refs. [26,27] in the massless limit [10] with the defect scattering formalism [28], where free particles are reflected and transmitted by the impurity with respective amplitudes $\hat{R}(\omega)$ and $\hat{T}(\omega)$ depending on their energy ω . At leading order, one finds (see the Supplemental Material [29])

$$S(\ell, L) = -\frac{1}{4} \int_0^\infty \frac{d\omega}{\omega} e^{-2L\omega} \hat{T}(\omega)^2 - \frac{1}{8} \int_0^\infty \frac{d\omega}{\omega} [e^{-4\ell\omega} + e^{-4(L-\ell)\omega}] \hat{R}(\omega)^2. \quad (7)$$

The IR divergence of Eq. (7) in the $\omega \ll 1$ limit can be cured by Γ -function regularization techniques, or, more elegantly, by computing the logarithmic derivative (5).

In the weak link case, one has $\hat{T}(\omega)^2 = \cos^2 \xi$ and $\hat{R}(\omega)^2 = \sin^2 \xi$. The parameter $\xi = (\pi/2) - 2 \arctan J'$ is independent of ω , since the perturbation is exactly marginal. In this case, $S(\ell, L)$ can, however, be computed exactly [5,30], so we turn instead to the more challenging dot case. (Note that the same formalism applies also to the weak link case with $\Delta = -(1/\sqrt{2})$, which is interacting on the lattice but can be refermionized in the scaling limit.) For the dot case at $\Delta = 0$, one has $\hat{T}(\omega)^2 = (T_B/(T_B + \omega))^2$ and $\hat{R}(\omega)^2 = (\omega/(T_B + \omega))^2$, and we stress that $\hat{R}^2 + \hat{T}^2 \neq 1$ only because unitarity has been broken by a Wick rotation in the computation leading to Eq. (7). Our main result is then the lowest-order FF approximation to the EE scaling function (5):

$$f(x, \alpha) = \frac{2}{3} \int_0^\infty dv e^{-2v} \left(\frac{x}{x+v} \right)^2 + \frac{2}{3} \int_0^\infty dv \left(\frac{\alpha}{e^{4\alpha v}} + \frac{1-\alpha}{e^{4(1-\alpha)v}} \right) \left(\frac{v}{x+v} \right)^2 \quad (8)$$

with the scaling variables $x = LT_B$ and $\alpha = \ell/L$. Notice that we have multiplied the actual result of the computation by a factor $4/3$ in order to obtain the correct UV and IR limits [10]. This renormalization is justified, e.g., by noticing that resummation of the full FF expansion in the UV/IR reproduces [10,27] the known CFT result

[24,25]. We note that the high-energy $x \ll 1$ expansion of Eq. (8) contains an $x \ln x$ term for all α , thus illustrating the nonperturbative nature of the EE, as already noticed for $\alpha = 1/2$ in Ref. [10].

From Eq. (8) we also obtain the FF approximation to the MI scaling function:

$$\frac{\partial I(L)}{\partial \ln L} \equiv g(x) = \frac{4}{3} \int_0^\infty dv \left(\frac{e^{2v} - 1}{e^{4v}} \right) \left(\frac{x}{v+x} \right)^2. \quad (9)$$

In general the MI is only an upper bound on the entanglement between the two wires, but in the limits $g(0) = 0$ and $g(\infty) = 1/3$ the bound is seen to saturate.

Numerical results.—The EE scaling function $f(x, \alpha)$ exhibits a rich, nonmonotonic behavior in both variables (see Fig. 2), with an especially singular—and physically interesting—limit $\alpha \rightarrow 0$. We now check the accuracy of the FF approximation (8) against extensive numerics on the XX spin chain ($\Delta = 0$) with two weak links. Mapping the problem onto free fermions [31], the reduced density matrix can be obtained by diagonalizing the correlation matrix $\langle c_n^\dagger c_m \rangle$ [32], which in turn can be computed exactly from one-particle eigenstates (see the Supplemental Material [29]). To avoid numerical instabilities, we used both double and 50-digit numerical precision. Our largest computations, with two wires of $N = 32\,000$ sites each, are shown in Fig. 2. To avoid boundary effects, we considered intervals of length $L < N/10$. The values of S showed strong parity effects in L , which were attenuated by averaging data for L and $L+1$.

Discussion.—The agreement between the FF approximation (8) to $f(x, \alpha)$ and the numerics is excellent, extending to more than 5 decades in $x = LT_B$ and all values of α , including the $\alpha \rightarrow 0$ limit. Note that our results agree without any free parameter, as the scale $T_B = (J')^2$ can be computed exactly for $\Delta = 0$. The considerable qualitative differences between $\alpha = 1/100$ and $\alpha = 0$ are well reproduced by the numerics. (The case $\alpha = 0$ is realized numerically by letting the interval start at the quantum dot site.) The scaling collapse for different values of J' is remarkable, except for very small x (high energy) where the lattice discretization is manifest. Presumably the small remnant discrepancies with Eq. (8) would disappear by taking the FF computation to the next order (see Ref. [10] in the $\alpha = 1/2$ case).

More importantly, our universal scaling prediction (5) goes beyond free-fermion systems—or interacting systems that can be mapped onto free-fermions at low energy, and provides the correct description of entanglement in quantum impurity systems characterized by a Kondo temperature T_B . It would be very interesting to generalize this prediction to nonequilibrium setups, for example in the context of quantum quenches [33].

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